Original ARMA(2,4) model with mean switching

$$Y_t - \mu_{st} = \phi_1(Y_{t-1} - \mu_{st-1)} + \phi_2(Y_{t-2} - \mu_{st-2)} + e_t + \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \gamma_3 e_{t-3} + \gamma_4 e_{t-4}$$

Let $Y_t^* = Y_t - \mu_{st}$,

$$Y_t^* = \phi_1 Y_{t-1}^* + \phi_2 Y_{t-2}^* + e_t + \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \gamma_3 e_{t-3} + \gamma_4 e_{t-4}$$

$$(1 - \emptyset_1 L - \emptyset_2 L^2) Y_t^* = e_t + \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \gamma_3 e_{t-3} + \gamma_4 e_{t-4}$$

$$Y_t^* = \frac{1}{(1 - \emptyset_1 L - \emptyset_2 L^2)} [e_t + \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \gamma_3 e_{t-3} + \gamma_4 e_{t-4}]$$

Finally,

$$Y_t = \frac{1}{(1 - \emptyset_1 L - \emptyset_2 L^2)} [e_t + \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \gamma_3 e_{t-3} + \gamma_4 e_{t-4}] + \mu_{st}$$

State-space representation

Measurement:

$$Y_t = [1.0 \ \ \gamma_1 \ \ \gamma_2 \ \ \gamma_3 \ \ \gamma_4 \ ] \begin{bmatrix} \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \\ \beta_{4,t} \\ \beta_{5,t} \end{bmatrix} + \mu_{s,t}$$

Transition:

$$\begin{bmatrix} \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \\ \beta_{4,t} \\ \beta_{5,t} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \beta_{3,t-1} \\ \beta_{4,t-1} \\ \beta_{5,t-1} \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Where $\beta_{1,t} = (\frac{1}{1-\phi L - \phi L^2}) e_t$, $\beta_{2,t} = (\frac{1}{1-\phi L - \phi L^2}) e_{t-1}$, $\beta_{3,t} = (\frac{1}{1-\phi L - \phi L^2}) e_{t-2}$, $\beta_{4,t} = (\frac{1}{1-\phi L - \phi L^2}) e_{t-3}$, $\beta_{5,t} = (\frac{1}{1-\phi L - \phi L^2}) e_{t-4}$