The Asymmetric Effects of Oil Price Shocks*

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Abstract

In this paper I investigate the effects of oil price uncertainty and its asymmetry on real economic activity in the United States, in the context of a general bivariate framework in which a vector autoregression is modified to accommodate GARCH-in-Mean errors. The model allows for the possibilities of spillovers and asymmetries in the variance-covariance structure for real output growth and the change in the real price of oil. The measure of oil price uncertainty is the conditional variance of the oil price change forecast error. I isolate the effects of volatility in the change in the price of oil and its asymmetry on output growth and employ simulation methods to calculate Generalized Impulse Response Functions (GIRFs) and Volatility Impulse Response Functions (VIRFs) to trace the effects of independent shocks on the conditional means and the conditional variances, respectively, of the variables. I find that oil price uncertainty has a negative effect on output, and that shocks to the price of oil and its uncertainty have asymmetric effects on output.

JEL classification: E32, C32,

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1 Introduction

Questions regarding the relationship between the price of oil and economic activity are fundamental empirical issues in macroeconomics. Hamilton (1983) showed that oil prices had significant predictive content for real economic activity in the United States prior to 1972 while Hooker (1996) argued that the estimated linear relations between oil prices and economic activity appear much weaker after 1973. In the debate that followed, several authors have suggested that the apparent weakening of the relationship between oil prices and economic activity is illusory, arguing instead that the true relationship between oil prices and real economic activity is asymmetric, with the correlation between oil price decreases and output significantly different than the correlation between oil price increases and output — see, for example, Mork (1989) and Hamilton (2003). More recently, however, Edelstein and Kilian (2007, 2008) evaluate alternative hypotheses and argue that the evidence of asymmetry cited in the literature is driven by a combination of ignoring the effects of the 1986 Tax Reform Act on fixed investment and the aggregation of energy and non-energy related investment.

Although there exists a vast literature that investigates the effects of oil prices on the real economy, there are relatively few studies that investigate the effects of uncertainty about oil prices. Lee *et al.* (1995) were the first to employ recent advances in financial econometrics and model oil price uncertainty using a univariate GARCH (1,1) model. They calculated an oil price shock variable, reflecting the unanticipated component as well as the time-varying conditional variance of oil price changes, introduced it in various vector autoregression (VAR) systems, and found that oil price volatility is highly significant in explaining economic growth. They also found evidence of asymmetry, in the sense that positive shocks have a strong effect on growth while negative shocks do not. A disadvantage of the Lee *et al.* (1995) approach, however, is that oil price volatility is a generated regressor, as described by Pagan (1984).

In this paper I move the empirical literature forward, by investigating the asymmetric effects of uncertainty on output growth and oil price changes as well as the response of uncertainty about output growth and oil price changes to shocks. In doing so, I use an extremely general bivariate framework in which a vector autoregression is modified to accommodate GARCH-in-Mean errors, as detailed in Engle and Kroner (1995), Grier *et al* (2004), and Shields *et al.* (2005). The model allows for the possibilities of spillovers and asymmetries in the variance-covariance structure for real activity and the real price of oil. As in Elder and Serletis (2009), my measure of oil price change volatility is the conditional variance of the oil price change forecast error. I isolate the effects of oil price change volatility and its asymmetry on output growth and, following Koop *et al.* (1996), Grier *et al* (2004), and Hafner and Herwartz (2006), I employ simulation methods to calculate Generalized Impulse Response Functions (GIRFs) and Volatility Impulse Response Functions (VIRFs) to trace the effects of independent shocks on the conditional means and the conditional variances, respectively, of the variables.

I find that my bivariate, GARCH-in-mean, asymmetric VAR-BEKK model embodies a reasonable description of the monthly U.S. data, over the period from 1981:1 to 2007:1. I show that the conditional variance-covariance process underlying output growth and the change in the real price of oil exhibits significant non-diagonality and asymmetry, and present evidence that increased uncertainty about the change in the real price of oil is associated with a lower average growth rate of real economic activity. Generalized impulse response experiments highlight the asymmetric effects of positive and negative shocks in the change in the real price of oil to output growth. Also, volatility impulse response experiments reveal that the effect of good news (negative shocks to the change in the real price of oil) on the conditional variance of the change in the real price of oil differs in magnitude and persistence from that of bad news of similar magnitude. This result suggests that, given the relationship between oil price volatility and output, the asymmetric response of oil price volatility to oil price shocks might be a contributing factor in explaining the asymmetric relationship between oil prices and economic activity.

The paper is organized as follows. Section 3.2 presents the data and Section 3.3 provides a brief description of the bivariate, GARCH-in-Mean, asymmetric VAR-BEKK model. Sections 3.4, 3.5, and 3.6 assess the appropriateness of the econometric methodology by various information criteria and present and discuss the empirical results. The final section concludes the paper.

2 The Data

I use monthly data for the United States, from the Federal Reserve Economic Database (FRED) maintained by the Federal Reserve Bank of St. Louis, over the period from 1981:1 to 2007:1, on two variables — the industrial production index (y_t) and the real price of oil (oil_t). In particular, I use the spot price on West Texas Intermediate (WTI) crude oil as the nominal price of oil and divide it by the consumer price index (CPI) to obtain the real price of oil. Following Bernanke *et al.* (1997), Lee and Ni (2002), Hamilton and Herrera (2004), and Edelstein and Kilian (2008), I use the industrial production index as a proxy variable for real output. It is to be noted that industrial output reflects only manufacturing, mining, and utilities, and represents only about 20% of total output. It captures, however, economic activity that is likely to be directly affected by oil prices and uncertainty about oil prices.

Table 1 presents summary statistics for the annualized logarithmic first differences of y_t

and oil_t, denoted as $\Delta \ln y_t$ and $\Delta \ln oil_t$, and Figures 1 and 2 plot the $\ln y_t$ and $\Delta \ln y_t$ and $\ln oil_t$ and $\Delta \ln oil_t$ series, respectively, with shaded area indicating NBER recessions. Both $\Delta \ln y_t$ and $\Delta \ln oil_t$ are skewed and there is significant amount of excess kurtosis present in the data. Moreover, a Jarque-Bera (1980) test for normality, distributed as a $\chi^2(2)$ under the null hypothesis of normality, suggests that each of $\Delta \ln y_t$ and $\Delta \ln oil_t$ fails to satisfy the null hypothesis of the test.

A battery of unit root and stationarity tests are conducted in Table 1 in $\Delta \ln y_t$ and $\Delta \ln oil_t$. In particular, I report the augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1981)] and, given that unit root tests have low power against relevant trend stationary alternatives, I also present Kwiatkowski *et al.* (1992) tests, known as KPSS tests, for level and trend stationarity. As can be seen, the null hypothesis of a unit root can be rejected at conventional significance levels. Moreover, the t-statistics $\widehat{\eta}_\mu$ and $\widehat{\eta}_\tau$ that test the null hypotheses of level and trend stationarity are small relative to their 5% critical values of 0.463 and 0.146 (respectively), given in Kwiatkowski *et al.* (1992). I thus conclude that $\Delta \ln y_t$ and $\Delta \ln oil_t$ are stationary [integrated of order zero, or I(0), in the terminology of Engle and Granger (1987)].

In panel C of Table 1, I conduct Ljung-Box (1979) tests for serial correlation in $\Delta \ln y_t$ and $\Delta \ln oil_t$. The Q-statistics, $Q(4)$ and $Q(12)$, are asymptotically distributed as $\chi^2(36)$ on the null hypothesis of no autocorrelation. Clearly, there is significant serial dependence in the data. I also present (in the last column of panel C) Engle's (1982) ARCH χ^2 test statistic, distributed as a $\chi^2(1)$ on the null of no ARCH. The test indicates that there is strong evidence of conditional heteroscedasticity in each of the $\Delta \ln y_t$ and $\Delta \ln oil_t$ series.

Finally, as I am interested in the asymmetry of the volatility response to news, in panel D of Table 1 I present Engle and Ng (1993) tests for 'sign bias,' 'negative size bias,' and 'positive size bias,' based on the following regression equations, respectively,

$$\widehat{\varepsilon}_t^2 = \phi_0 + \phi_1 D_{t-1}^- + \xi_t; \tag{1}$$

$$\widehat{\varepsilon}_t^2 = \phi_0 + \phi_1 D_{t-1}^- \widehat{\varepsilon}_{t-1} + \xi_t; \tag{2}$$

$$\widehat{\varepsilon}_t^2 = \phi_0 + \phi_1 D_{t-1}^+ \widehat{\varepsilon}_{t-1} + \xi_t, \tag{3}$$

where $\widehat{\varepsilon}_t$ is the residual from a fourth-order autoregression of the raw data ($\Delta \ln y_t$ or $\Delta \ln oil_t$), treated as a collective measure of news at time t, D_{t-1}^- is a dummy variable that takes a value of one when $\widehat{\varepsilon}_{t-1}$ is negative (bad news) and zero otherwise, $D_{t-1}^+ = 1 - D_{t-1}^-$, picking up the observations with positive innovations (good news), and ϕ_0 and ϕ_1 are parameters. The t-ratio of the ϕ_1 coefficient in each of regression equations (1)-(3) is defined as the test statistic.

The sign bias test in equation (1) examines the impact that positive and negative shocks have on volatility which is not predicted by the volatility model under consideration. In particular, if the response of volatility to shocks is asymmetric (that is, positive and negative shocks to $\widehat{\varepsilon}_{t-1}$ impact differently upon the conditional variance, $\widehat{\varepsilon}_t^2$), then ϕ_1 will be statistically significant. Irrespective of whether the response of volatiltiy to shocks is symmetric or asymmetric, the size (or magnitude) of the shock could also affect volatility. The negative size bias test in equation (2) focuses on the asymmetric effects of negative shocks (that is, whether small and large negative shocks to $\widehat{\varepsilon}_{t-1}$ impact differently upon the conditional variance, $\widehat{\varepsilon}_t^2$). In this case, D_{t-1}^- is used as a slope dummy variable in equation (2) and negative size bias is present if ϕ_1 is statistically significant. The positive size bias test in equation (3) focuses on the different effects that large and small positive shocks have on volatility, and positive size bias is present if ϕ_1 is statistically significant in (3). I also conduct a joint test for both sign and size bias using the following regression equation,

$$\widehat{\varepsilon}_t^2 = \phi_0 + \phi_1 D_{t-1}^- + \phi_2 D_{t-1}^- \widehat{\varepsilon}_{t-1} + \phi_3 D_{t-1}^+ \widehat{\varepsilon}_{t-1} + \xi_t. \tag{4}$$

In the joint test in equation (4), the test statistic is equal to $T \times R^2$ (where R^2 is the R-squared from the regression) and follows a χ^2 distribution with three degrees of freedom under the null hypothesis of no asymmetric effects.

As can be seen in panel D of Table 1, the conditional volatility of output growth is sensitive to the sign and size of the innovation. In particular, there is strong evidence of sign and negative size bias in the output growth volatility, and the joint test for both sign and size bias is highly significant. Also, the conditional volatility of the change in the price of oil displays negative size bias and the joint test for both sign and size bias is significant at conventional significance levels.

3 Econometric Methodology

Given the evidence of conditional heteroscedasticity in the $\Delta \ln y_t$ and $\Delta \ln oil_t$ series, I characterize the joint data generating process underlying $\Delta \ln y_t$ and $\Delta \ln oil_t$ as a bivariate

GARCH-in-Mean model, as follows

$$\boldsymbol{y}_t = \mathbf{a} + \sum_{i=1}^{p} \boldsymbol{\Gamma}_i \boldsymbol{y}_{t-i} + \sum_{j=0}^{q} \boldsymbol{\Psi}_j \sqrt{\boldsymbol{h}_{t-j}} + \boldsymbol{e}_t \tag{5}$$

$$\boldsymbol{e}_t | \, \Omega_{t-1} \sim (\mathbf{0}, \, \boldsymbol{H}_t), \qquad \boldsymbol{H}_t = \begin{bmatrix} h_{\Delta \ln y \Delta \ln y, t} & h_{\Delta \ln y \Delta \ln oil, t} \\ & \\ h_{\Delta \ln oil \Delta \ln y, t} & h_{\Delta \ln oil \Delta \ln oil, t} \end{bmatrix},$$

where $\mathbf{0}$ is the null vector, Ω_{t-1} denotes the available information set in period $t-1$, and

$$\boldsymbol{y}_t = \begin{bmatrix} \Delta \ln y_t \\ \Delta \ln oil_t \end{bmatrix}; \boldsymbol{e}_t = \begin{bmatrix} e_{\Delta \ln y, t} \\ e_{\Delta \ln oil, t} \end{bmatrix}; \; \boldsymbol{h}_t = \begin{bmatrix} h_{\Delta \ln y \Delta \ln y, t} \\ h_{\Delta \ln oil \Delta \ln oil, t} \end{bmatrix};$$

$$\mathbf{a} = \begin{bmatrix} a_{\Delta \ln y} \\ a_{\Delta \ln oil} \end{bmatrix}; \; \boldsymbol{\Gamma}_i = \begin{bmatrix} \gamma_{11}^{(i)} & \gamma_{12}^{(i)} \\ & \\ \gamma_{21}^{(i)} & \gamma_{22}^{(i)} \end{bmatrix}; \; \boldsymbol{\Psi}_j = \begin{bmatrix} \psi_{11}^{(j)} & \psi_{12}^{(j)} \\ & \\ \psi_{21}^{(j)} & \psi_{22}^{(j)} \end{bmatrix}.$$

Notice that I have not added any error correction term in the model as the null hypothesis of no cointegration between output ($\ln y_t$) and the real price of oil ($\ln oil_t$) cannot be rejected.

Multivariate GARCH models require that I specify volatilities of $\Delta \ln y_t$ and $\Delta \ln oil_t$, measured by conditional variances. Several different specifications have been proposed in the literature, including the VECH model of Bollerslev *et al.* (1988), the CCORR model of Bollerslev (1990), the FARCH specification of Engle *et al.* (1990), the BEKK model proposed by Engle and Kroner (1995), and the DCC model of Engle (2002). However, none of these specifications is capable of capturing the asymmetry of the volatility response to news.

In this regard, given the asymmetric effects of news on volatility in the $\Delta \ln y_t$ and $\Delta \ln oil_t$ series, I use an asymmetric version of the BEKK model, introduced by Grier *et al.* (2004), as follows

$$\boldsymbol{H}_t = \boldsymbol{C}'\boldsymbol{C} + \sum_{j=1}^{f} \boldsymbol{B}_j'\boldsymbol{H}_{t-j}\boldsymbol{B}_j + \sum_{k=1}^{g} \boldsymbol{A}_k'\boldsymbol{e}_{t-k}\boldsymbol{e}_{t-k}'\boldsymbol{A}_k + \boldsymbol{D}'\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}' \tag{6}$$

where $\boldsymbol{C}$, $\boldsymbol{B}_j$, $\boldsymbol{A}_k$, and $\boldsymbol{D}$ are 2×2 matrices (for all values of j and k), with $\boldsymbol{C}$ being a triangular matrix to ensure positive definiteness of $\boldsymbol{H}$. In equation (6), $\boldsymbol{u}_t = (u_{\Delta \ln y,t}, u_{\Delta \ln oil,t})'$ and captures potential asymmetric responses. In particular, if the change in the price of oil, $\Delta \ln oil_t$, is higher than expected, I take that to be bad news. I therefore capture bad news about oil price changes by a positive oil price change residual, by defining $u_{\Delta \ln oil,t} = \max\{e_{\Delta \ln oil,t}, 0\}$. I also capture bad news about output growth by defining $u_{\Delta \ln y,t} = \min\{e_{\Delta \ln y,t}, 0\}$. Hence, $\boldsymbol{u}_t = (u_{\Delta \ln y,t}, u_{\Delta \ln oil,t})' = (\min\{e_{\Delta \ln y,t}, 0\}, \max\{e_{\Delta \ln oil,t}, 0\})'$.

The specification in equation (6) allows past volatilities, $\boldsymbol{H}_{t-j}$, as well as lagged values of $\boldsymbol{e}\boldsymbol{e}'$ and $\boldsymbol{u}\boldsymbol{u}'$, to show up in estimating current volatilities of $\Delta \ln y_t$ and $\Delta \ln oil_t$. Moreover, the introduction of the $\boldsymbol{u}\boldsymbol{u}'$ term in (6) extends the BEKK model by relaxing the assumption of symmetry, thereby allowing for different relative responses to positive and negative shocks in the conditional variance-covariance matrix, $\boldsymbol{H}$.

There are $n + n^2(p+q) + n(n+1)/2 + n^2(f+g+1)$ parameters in (5)-(6) and in order to deal with estimation problems in the large parameter space I assume that $f = g = 1$ in equation (6), consistent with recent empirical evidence regarding the superiority of GARCH(1,1) models — see, for example, Hansen and Lunde (2005). It is also to be noted that I have not included an interest rate variable in the model (in the $\boldsymbol{y}_t$ equation), although it would seem to be important as oil prices affect output through an indirect effect on the rate of interest. I have kept the dimension of the model low because of computational and degree of freedom problems in the large parameter space. For example, with $n = 2$, $p = q = 2$ in equation (5) and $f = g = 1$ in equation (6), the model has 33 parameters to be estimated. If I introduce one more variable in the model (like the interest rate), then I would have to estimate 81 parameters. Moreover, the tests that I conduct in Section 4 indicate that the exclusion of such a variable is not expected to result in significant misspecification error.

In order to estimate my bivariate GARCH-in-Mean asymmetric BEKK model, I construct the likelihood function, ignoring the constant term and assuming that the statistical innovations are conditionally gaussian

$$l_t = -\frac{1}{2}\sum_{t=t+1}^{T} \log|\boldsymbol{H}_t| - \frac{1}{2}\sum_{t=t+1}^{T}\left(\boldsymbol{e}_t^{\prime}\boldsymbol{H}_t^{-1}\boldsymbol{e}_t\right),$$

where $\boldsymbol{e}_t$ and $\boldsymbol{H}_t$ are evaluated at their estimates. The log-likelihood is maximized with respect to the parameters $\boldsymbol{\Gamma}_i$ $(i = 1, \cdots, p)$, Ψ_j $(j = 1, \cdots, q)$, $\boldsymbol{C}$, $\boldsymbol{B}$, $\boldsymbol{A}$, and $\boldsymbol{D}$. As I am using the BEKK model, I do not need to impose any restrictions on the variance parameters to make $\boldsymbol{H}_t$ positive definite. Moreover, I am estimating all the parameters simultaneously rather than estimating mean and variance parameters separately, thus avoiding the Lee *et al.* (1995) problem of generated regressors.

4 Empirical Evidence

Initially I used the AIC and SIC criteria to select the optimal values of p and q in (5). However, because of computational difficulties in the large parameter space and remaining serial correlation and ARCH effects in the standardized residuals, I set $p = 3$ and $q = 1$ in equation (5). Hence, with $p = 3$ and $q = 1$ in equation (5), and $f = g = 1$ in equation (6), I estimate a total of 37 parameters. Maximum likelihood (ML) estimates of the parameters and diagnostic test statistics are presented in Tables 2 and 3.

I conduct a battery of misspecification tests, using robustified versions of the standard test statistics based on the standardized residuals,

$$z_i = \frac{e_{i,t}}{\sqrt{\widehat{h}_{ij,t}}}, \qquad \text{for } i, j = \Delta \ln y,\, \Delta \ln oil.$$

As shown in panel A of Table 2, the Ljung-Box Q-statistics for testing serial correlation cannot reject the null hypothesis of no autocorrelation (at conventional significance levels) for the values and the squared values of the standardized residuals, suggesting that there is no evidence of conditional heteroscedasticity. Moreover, the failure of the data to reject the null hypotheses of $E(z) = 0$ and $E(z^2) = 1$, implicitly indicates that our bivariate asymmetric GARCH-in-Mean model does not bear significant misspecification error — see, for example, Kroner and Ng (1998).

In Table 3, I also present diagnostic tests suggested by Engle and Ng (1993) and Kroner and Ng (1998), based on the 'generalized residuals,' defined as $e_{i,t}e_{j,t} - h_{ij,t}$ for $i, j = \Delta \ln y$, $\Delta \ln oil$. For all symmetric GARCH models, the news impact curve — see Engle and Ng (1993) — is symmetric and centered at $e_{i,t-1} = 0$. A generalized residual can be thought of as the distance between a point on the scatter plot of $e_{i,t}e_{j,t}$ from a corresponding point on the news impact curve. If the conditional heteroscedasticity part of the model is correct, $E_{t-1}(e_{i,t}e_{j,t} - h_{ij,t}) = 0$ for all values of i and j, generalized residuals should be uncorrelated with all information known at time $t - 1$. In other words, the unconditional expectation of $e_{i,t}e_{j,t}$ should be equal to its conditional one, $h_{ij,t}$.

The Engle and Ng (1993) and Kroner and Ng (1998) misspecification indicators test whether I can predict the generalized residuals by some variables observed in the past, but which are not included in the model — this is exactly the intuition behind $E_{t-1}(e_{i,t}e_{j,t} - h_{ij,t}) = 0$. In this regard, I follow Kroner and Ng (1998) and Shields *et al.* (2005) and define two sets of misspecification indicators. In a two dimensional space, I first partition ($e_{\Delta \ln y,t-1}$, $e_{\Delta \ln oil,t-1}$) into four quadrants in terms of the possible sign of the two residuals. Then, to shed light on any possible sign bias of the model, I define the first set of indicator functions as $I(e_{\Delta \ln y,t-1} < 0)$, $I(e_{\Delta \ln oil,t-1} < 0)$, $I(e_{\Delta \ln y,t-1} < 0;\ e_{\Delta \ln oil,t-1} < 0)$, $I(e_{\Delta \ln y,t-1} > 0;\ e_{\Delta \ln oil,t-1} < 0)$, $I(e_{\Delta \ln y,t-1} < 0;\ e_{\Delta \ln oil,t-1} > 0)$ and $I(e_{\Delta \ln y,t-1} > 0;\ e_{\Delta \ln oil,t-1} > 0)$, where $I(\cdot)$ equals one if the argument is true and zero otherwise. Significance of any of these indicator functions indicates that the model (5)-(6) is incapable of predicting the effects of some shocks to either $\Delta \ln y_t$ or $\Delta \ln oil_t$. Moreover, due to the fact that the possible effect of a shock could be a function of both the size and the sign of the shock, I define a second set of indicator functions, $e^2_{\Delta \ln y_{t-1}} I(e_{\Delta \ln y,t-1} < 0)$, $e^2_{\Delta \ln y,t-1} I(e_{\Delta \ln oil,t-1} < 0)$, $e^2_{\Delta \ln oil,t-1} I(e_{\Delta \ln y,t-1} < 0)$, and $e^2_{\Delta \ln oil,t-1} I(e_{\Delta \ln oil,t-1} < 0)$. These indicators are technically scaled versions of the former ones, with the magnitude of the shocks as a scale measure.

I conducted indicator tests and report the results in Table 3. As can be seen in Table 3, most of the indicators fail to reject the null hypothesis of no misspecification — all test statistics in Table 3 are distributed as $\chi^2(1)$. Hence, our model (5)-(6) captures the effects of all sign bias and sign-size scale depended shocks in predicting volatility and there is no significant misspecification error. This means that the exclusion of the interest rate variable (in $\boldsymbol{y}_t$), mentioned earlier, is not expected to lead to significant misspecification problems.

Turning now back to panel B of Table 2, the diagonality restriction, $\gamma^{(i)}_{12} = \gamma^{(i)}_{21} = 0$ for $i = 1, 2, 3$, is rejected, meaning that the data provide strong evidence of the existence of dynamic interactions between $\Delta \ln y_{t-1}$ and $\Delta \ln oil_t$. The null hypothesis of homoscedastic disturbances requires the $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$ matrices to be jointly insignificant (that is, $\alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$ for all i, j) and is rejected at the 1% level or better, suggesting that there is significant conditional heteroscedasticity in the data. The null hypothesis of symmetric conditional variance-covariances, which requires all elements of the $\boldsymbol{D}$ matrix to be jointly insignificant (that is, $\delta_{ij} = 0$ for all i, j), is rejected at the 1% level or better, implying the existence of some asymmetries in the data which the model is capable of capturing. Also, the null hypothesis of a diagonal covariance process requires the off-diagonal elements of the $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$ matrices to be jointly insignificant (that is, $\alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$), but these estimated coefficients are jointly significant at the 14% level of significance.

Thus the $\Delta \ln y_t$-$\Delta \ln oil_t$ process is strongly conditionally heteroscedastic, with innovations to oil price changes significantly influencing the conditional variance of output growth in an asymmetric way. Moreover, the sign as well as the size of oil price change innovations are important. To establish the relationship between the volatility in the change in the price of oil and output growth, in Table 2 I test the null hypothesis that the volatility of $\Delta \ln oil_t$ does not (Granger) cause output growth, $H_0 : \psi_{12} = 0$. I strongly reject the null hypothesis, finding strong evidence in support of the hypothesis that $\Delta \ln oil_t$ volatility Granger causes output growth.

In Figures 3, 4, and 5 I plot the conditional standard deviations of output growth and the change in the price of oil as well as the conditional covariance implied by our estimates of the asymmetric VAR-BEKK model in Table 2. In Figure 3, the biggest episode of output growth volatility coincides with the 1982 NBER recession — the biggest recession in the sample. Regarding the change in the real price of oil, $\Delta \ln Oil_t$, Figure 4 shows that the biggest episodes of oil price change volatility took place in 1982, 1986, 1990, and 1999. All of these volatility jumps in $\Delta \ln Oil_t$ do not coincide with NBER recessions, except perhaps that in 1991, but the relatively smaller volatility jump in the oil price change in 1982 coincides with the biggest recession in the sample. The volatility jumps in 1986 and 1999 are the results of steep oil price declines rather than oil price increases. Finally, the conditional covariance between $\Delta \ln y_t$ and $\Delta \ln Oil_t$, shown in Figure 5, is highest in 1986, 1990, and 1999, and is negative in 1982 and 2005.

5 Generalized Impulse Response Functions

As van Dijk *et al.* (2007) recently put it, "it generally is difficult, if not impossible, to fully understand and interpret nonlinear time series models by considering the estimated values of the model parameters only." Thus, in order to quantify the dynamic response of output growth and oil price changes to shocks and to investigate the statistical significance of the asymmetry in the variance-covariance structure, I calculate Generalized Impulse Response Functions (GIRFs), introduced by Koop *et al.* (1996) and recently used by Grier *et al.* (2004), based on our bivariate, GARCH-in-Mean, asymmetric VAR-BEKK model (5)-(6).

Traditional impulse response functions, which are more usefully applied to linear models than to nonlinear ones, measure the effect of a shock (say of size δ) hitting the system at time t on the state of the system at time $t+n$, given that no other shocks hit the system. As Koop *et al.* (1996, p. 121) put it, "the idea is very similar to Keynesian multiplier analysis, with the difference that the analysis is carried out with respect to shocks or 'innovations' of macroeconomic time series, rather than the series themselves (such as investment or government expenditure)." In the case of multivariate nonlinear models, however, traditional impulse response functions depend on the sign and size of the shock as well as the history of the system (i.e., expansionary or contractionary) before the shock hits — see, for example, Potter (2000).

In my asymmetric bivariate, GARCH-in-Mean, VAR-BEKK model, shocks impact on output growth and the change in the price of oil through the conditional mean as described in equation (5) and with lags through the conditional variance as described in equation (6). Moreover, the impulse responses of $\Delta \ln y_t$ and $\Delta \ln Oil_t$ depend on the composition of the $e_{\Delta \ln y_t}$ and $e_{\Delta \ln oil_t}$ shocks — that is, the effect of a shock to $\Delta \ln Oil_t$ is not isolated from having a contemporaneous effect on $\Delta \ln y_t$ and vice versa. The GIRFs that I use in this paper provide a method of dealing with the problems of shock, history, and composition dependence of impulse responses in multivariate (linear and) nonlinear models.

In particular, assuming that $\boldsymbol{y}_t$ is a random vector, the GIRF for an arbitrary current shock, $\boldsymbol{v}_t$, and history, ω_{t-1}, is defined as

$$\text{GIRF}_{\boldsymbol{y}}(n, \boldsymbol{v}_t, \omega_{t-1}) = E\left[\boldsymbol{y}_{t+n}|\boldsymbol{v}_t, \omega_{t-1}\right] - E\left[\boldsymbol{y}_{t+n}|\omega_{t-1}\right], \tag{7}$$

for $n = 0, 1, 2, \cdots$. Assuming that $\boldsymbol{v}_t$ and ω_{t-1} are realizations of the random variables $\boldsymbol{V}_t$ and Ω_{t-1} (where Ω_{t-1} is the set containing information used to forecast $\boldsymbol{y}_t$) that generate realizations of $\{\boldsymbol{y}_t\}$, then according to Koop *et al.* (1996), the GIRF in (7) can be considered to be a realization of a random variable defined by

$$\text{GIRF}_{\boldsymbol{y}}(n, \boldsymbol{V}_t, \Omega_{t-1}) = E\left[\boldsymbol{y}_{t+n}|\boldsymbol{V}_t, \Omega_{t-1}\right] - E\left[\boldsymbol{y}_{t+n}|\Omega_{t-1}\right]. \tag{8}$$

Equation (8) is the difference between two conditional expectations, $E\left[\boldsymbol{y}_{t+n}|\boldsymbol{V}_t, \Omega_{t-1}\right]$ and $E\left[\boldsymbol{y}_{t+n}|\Omega_{t-1}\right]$, which are themselves random variables. Hence, $\text{GIRF}_{\boldsymbol{y}}(n, \boldsymbol{V}_t, \Omega_{t-1})$, represents a realization of this random variable.

The computation of GIRFs in the case of multivariate nonlinear models is made difficult by the inability to construct analytical expressions for the conditional expectations, $E\left[\boldsymbol{y}_{t+n}|\boldsymbol{V}_t,\Omega_{t-1}\right]$ and $E\left[\boldsymbol{y}_{t+n}|\Omega_{t-1}\right]$, in equation (8). To deal with this problem, Monte Carlo methods of stochastic simulation are used to construct the GIRFs. Here, I allow for time-varying composition dependence and follow the algorithm described in Koop *et al.* (1996). In particular, using 310 data points as histories, I first transform the estimated residuals by using the variance-covariance structure and Jordan decomposition. Then at each history, 50 realizations are drawn randomly, thereby obtaining identical and independent distributions over time. Recovering the time varying dependence among the residuals, 15500 realizations of impulse responses are calculated for each horizon. Finally, the whole process is replicated 150 times to average out the effects of impulses.

The GIRFs to an average shock in $\Delta\ln y_t$ and $\Delta\ln oil_t$ are shown in Figures 6 and 7 — they show the effect on $\Delta\ln y_t$ and $\Delta\ln oil_t$ of an average initial shock in $\Delta\ln y_t$ and $\Delta\ln oil_t$. As can be seen, none of the shocks is very persistent, although the shock to $\Delta\ln oil_t$ on $\Delta\ln y_t$ is more persistent than the shock to $\Delta\ln y_t$ on $\Delta\ln oil_t$, as it takes longer for $\Delta\ln y_t$ to return to its original value. Shocks to output growth and the change in the price of oil provide a large stimulus to $\Delta\ln y_t$ and $\Delta\ln oil_t$ for the first few months. In particular, in response to an oil price change shock, output growth declines by more than 0.75% in the first quarter of the year and returns to its mean within one and a half years. Also the change in the price of oil responds very strongly (almost around 12%), to the innovation in output growth within the first few months of the year.

In Figures 8 and 9, I differentiate between positive and negative shocks, in order to address issues regarding the asymmetry of shocks. As can be seen in Figure 8, output growth declines due to a positive $\Delta\ln oil_t$ shock and increases in response to a negative $\Delta\ln oil_t$ shock. The responses are not the mirror image of each other, suggesting that output growth, $\Delta \ln y_t$, responds asymmetrically to shocks in the change in the price of oil, $\Delta \ln oil_t$. Also the response of output growth to a negative $\Delta \ln oil_t$ shock returns to zero faster than to a positive shock of equal magnitude, suggesting that positive shocks in the change in the price of oil have more persistent effects on output growth than negative ones. Figure 9 shows the GIRFs of $\Delta \ln oil_t$ to positive and negative output growth shocks. A positive $\Delta \ln y_t$ shock raises $\Delta \ln oil_t$ and a negative $\Delta \ln y_t$ shock lowers $\Delta \ln oil_t$. The response of $\Delta \ln oil_t$ to output growth shocks is also asymmetric — a positive output growth shock has a larger effect on the change in the price of oil compared to a negative $\Delta \ln y_t$ shock.

Given the asymmetric nature of the specification of our bivariate asymmetric VAR-BEKK model, we follow Van Dijk *et al.* (2007) and use the GIRFs to positive and negative shocks to compute a random asymmetry measure, defined as follows,

$$\mathrm{ASY}_{\boldsymbol{y}}(n, \boldsymbol{V}_t^+, \Omega_{t-1}) = \mathrm{GIRF}_{\boldsymbol{y}}(n, \boldsymbol{V}_t^+, \Omega_{t-1}) + \mathrm{GIRF}_{\boldsymbol{y}}(n, -\boldsymbol{V}_t^+, \Omega_{t-1}), \tag{9}$$

where $\mathrm{GIRF}_{\boldsymbol{y}}(n, \boldsymbol{V}_t^+, \Omega_{t-1})$ denotes the GIRF derived from conditioning on the set of all possible positive shocks, $\mathrm{GIRF}_{\boldsymbol{y}}(n, -\boldsymbol{V}_t^+, \Omega_{t-1})$ denotes the GIRF derived from conditioning on the set of all possible negative shocks, and $\boldsymbol{V}_t^+ = \{\boldsymbol{v}_t | \boldsymbol{v}_t > 0\}$. The distribution of $\mathrm{ASY}(n, \boldsymbol{V}_t^+, \Omega_{t-1})$ can provide an indication of the asymmetric effects of positive and negative shocks. In particular, if $\mathrm{ASY}(n, \boldsymbol{V}_t^+, \Omega_{t-1})$ has a symmetric distribution with a mean of zero, then positive and negative shocks have exactly the same effect (with opposite sign).

We have computed the asymmetry measures for $\Delta \ln y_t$ and $\Delta \ln oil_t$ and show the distributions of the respective $\mathrm{ASY}_{\boldsymbol{y}}(n, \boldsymbol{V}_t^+, \Omega_{t-1})$ measures in Figures 10 and 11 at horizons $n = 6$, $n = 9$, and $n = 12$. As can be seen in Figure 10, on average output growth exhibits more persistence to a positive $\Delta \ln oil_t$ shock than to a negative one. In particular, the loss of output growth due to a positive $\Delta \ln oil_t$ shock (bad news) at horizon $n = 9$ is 0.083% in excess of the gain in output growth from a negative $\Delta \ln oil_t$ shock (good news) of equal magnitude. Figure 11 shows the asymmetry measure for an output growth shock on $\Delta \ln oil_t$. We find a stronger effect of a positive output growth shock on the change in the price of oil than of a negative shock of equal magnitude. On average at horizon 9, the increase in $\Delta \ln oil_t$ due to a positive output growth shock is 0.256% in excess of the decrease in $\Delta \ln oil_t$ due to a negative $\Delta \ln y_t$ shock.

6 Volatility Impulse Response Functions

The GIRFs, introduced by Koop *et al.* (1996), trace the effects of independent shocks (or news) on the conditional mean. Recently, Hafner and Herwartz (2006) have introduced a new concept of impulse response functions, known as 'volatility impulse response functions' (VIRFs), tracing the effects of independent shocks on the conditional variance — see also Shields *et al.* (2005) for an early application of the Hafner and Herwartz (2006) VIRFs concept.

We start with the conditional variance-covariance matrix of $\boldsymbol{e}_t$, $\boldsymbol{H}_t$, and define $\boldsymbol{Q}_t = \text{vech}(\boldsymbol{H}_t)$ to be a 3×1 random vector with the following elements (in that order): $h_{\Delta \ln y,t}$, $h_{\Delta \ln y \Delta \ln oil,t}$, $h_{\Delta \ln oil,t}$. Then the VIRFs of $\boldsymbol{Q}_t$, for $n = 0, 1, \cdots$, are given by

$$\text{VIRF}_{\boldsymbol{Q}}(n, \boldsymbol{v}_t, \omega_{t-1}) = E\left[\boldsymbol{Q}_{t+n} | \boldsymbol{v}_t, \omega_{t-1}\right] - E\left[\boldsymbol{Q}_{t+n} | \omega_{t-1}\right]. \tag{10}$$

Hence, the VIRF is conditional on the initial shock and history, $\boldsymbol{v}_t$ and ω_{t-1}, and constructs the response by averaging out future innovations given the past and present. Following Koop *et al.* (1996) and assuming that $\boldsymbol{v}_t$ and ω_{t-1} are realizations of the random variables $\boldsymbol{V}_t$ and Ω_{t-1} that generate realizations of $\{\boldsymbol{Q}\}$, the VIRF in (10) can be considered to be a realization of a random variable given by

$$\mathrm{VIRF}_{\boldsymbol{Q}}(n, \boldsymbol{V}_t, \Omega_{t-1}) = E\left[\boldsymbol{Q}_{t+n}|\boldsymbol{V}_t, \Omega_{t-1}\right] - E\left[\boldsymbol{Q}_{t+n}|\Omega_{t-1}\right].$$

As already noted, the first element of $\mathrm{VIRF}_{\boldsymbol{Q}}(n, \boldsymbol{V}_t, \Omega_{t-1})$ gives the impulse response of the conditional variance of $\Delta \ln y_t$, $h_{\Delta \ln y,t}$, the second that of the conditional covariance, $h_{\Delta \ln y \Delta \ln oil,t}$, and the third that of the conditional variance of $\Delta \ln oil_t$, $h_{\Delta \ln oil,t}$. It should also be noted that in contrast to the GIRFs where positive and negative shocks produce opposite effects, VIRFs consist of the same (positive sign) effect irrespective of the sign of the shocks. Also, shock linearity does not hold in the case of VIRFs. Finally, unlike traditional impulse responses that do not depend on history, VIRFs depend on history through the conditional variance-covariance matrix at time $t = 0$ when the innovation occurs.

Using an analytic version of the VIRF, as described in Hafner and Herwartz (2006), I show the VIRFs to shocks in $\Delta \ln y_t$ and $\Delta \ln oil_t$ in Figures 12 and 13. In Figure 12 (the responses of oil price change volatility are on the secondary y-axis), shocks to the change in the price of oil, $\Delta \ln oil_t$, produce much higher responses in the conditional variance of the change in the price of oil, $h_{\Delta \ln oil,t}$, than in the conditional variance of output growth, $h_{\Delta \ln y,t}$. Moreover, the responses of both oil price change and output growth volatility are persistent — they take more than two years to return to zero. In Figure 13 (again the responses of oil price change volatility are on the secondary y-axis), shocks to output growth have also a very remarkable impact on the conditional variance of the change in the price of oil, $h_{\Delta \ln oil,t}$, and the conditional variance of output growth, $h_{\Delta \ln y,t}$. The peak response of the change in the price of oil is much lower than that of output growth volatility and $h_{\Delta \ln oil,t}$ takes longer to return to its original position compared to $h_{\Delta \ln y,t}$. In Figure 14, we differentiate again between positive and negative shocks in order to investigate the asymmetry of shocks on the conditional variance of the change in the real price of oil. As it is shown, negative oil price shocks have a larger impact on oil price change volatility than positive oil price shocks do.

As with the GIRFs, we use the VIRFs to positive and negative innovations to compute the following random asymmetry measure

$$\mathrm{ASY}_{\boldsymbol{Q}}(n, \boldsymbol{V}_t^+, \Omega_{t-1}) = \mathrm{VIRF}_{\boldsymbol{Q}}(n, \boldsymbol{V}_t^+, \Omega_{t-1}) - \mathrm{VIRF}_{\boldsymbol{Q}}(n, -\boldsymbol{V}_t^+, \Omega_{t-1}), \tag{11}$$

where $\mathrm{VIRF}_{\boldsymbol{Q}}(n, \boldsymbol{V}_t^+, \Omega_{t-1})$ denotes the VIRF derived from conditioning on the set of all possible positive innovations, $\mathrm{VIRF}_{\boldsymbol{Q}}(n, -\boldsymbol{V}_t^+, \Omega_{t-1})$ denotes the VIRF derived from conditioning on the set of all possible negative innovations, and $\boldsymbol{V}_t^+ = \{\boldsymbol{v}_t | \boldsymbol{v}_t > 0\}$. The distribution of $\mathrm{ASY}_{\boldsymbol{Q}}(n, \boldsymbol{V}_t^+, \Omega_{t-1})$ will be centered at zero if positive and negative shocks have exactly the same effect. The difference between the random asymmetry measures (9) and (11) is that the latter is the diffrence between $\mathrm{VIRF}_{\boldsymbol{Q}}(n, \boldsymbol{V}_t^+, \Omega_{t-1})$ and $\mathrm{VIRF}_{\boldsymbol{Q}}(n, -\boldsymbol{V}_t^+, \Omega_{t-1})$ whereas the former is the sum of $\mathrm{GIRF}_{\boldsymbol{y}}(n, \boldsymbol{V}_t^+, \Omega_{t-1})$ and $\mathrm{GIRF}_{\boldsymbol{y}}(n, -\boldsymbol{V}_t^+, \Omega_{t-1})$. This is so because unlike the GIRFs where positive and negative shocks cause the response functions to take opposite signs, the VIRFs are made up of the squares of the innovations and are thus of the same sign.

I have computed the asymmetry measures $\mathrm{ASY}_{\boldsymbol{Q}}(n, \boldsymbol{V}_t^+, \Omega_{t-1})$ and show the distributions of these measures in Figures 15 and 16, at horizon $n = 3, 6, 9$. The distribution in Figure 15 indicates that on average negative shocks to the change in the price of oil have more persistent effects on oil price change volatility than positive shocks do. The asymmetry measure for an oil price change shock to oil price change volatility at horizon $n = 3$ is -4.13%. Given the negative relationship between oil price change volatility and output growth, this asymmetric response of oil price change volatility tends to further weaken the short run response of ouput growth to lower oil prices. This result further helps in explaining the asymmetric response of output growth to oil price shocks. Also, in Figure 16, positive shocks to the change in the price of oil have more persistent effects on the volatility of output growth than negative shocks do. The asymmetry measure for oil price change shocks to the volatility in the change in the price of oil at horizon $n = 3$ is 0.13%.

7 Conclusion

Recent empirical research regarding the relationship between the price of oil and real economic activity has focused on the role of uncertainty about oil prices. In this paper, I examine the effects of oil price uncertainty and its asymmetry on real economic activity in the United States, in the context of a dynamic framework in which a vector autoregression has been modified to accommodate asymmetric GARCH-in-Mean errors. I use a bivariate VAR (in output growth and the change in the real price of oil) because the identification of higher order VARs is usually highly questionable.

My model is extremely general and allows for the possibilities of spillovers and asymmetries in the variance-covariance structure for real output growth and the change in the real price of oil. My measure of oil price uncertainty is the conditional variance of the oil price change forecast error. I isolate the effects of volatility in the change in the price of oil and its asymmetry on output growth and, following Koop *et al.* (1996), Hafner and Herwartz (2006), and van Dijk *et al.* (2007) I employ simulation methods to calculate Generalized Impulse Response Functions (GIRFs) and Volatility Impulse Response Functions (VIRFs) to trace the effects of independent shocks on the conditional mean and the conditional variance, respectively, of output growth and the change in the real price of oil.

I find that my bivariate, GARCH-in-Mean, asymmetric BEKK model embodies a reasonable description of the United States data on output growth and the change in the real price of oil. I show that the conditional variance-covariance process underlying output growth and the change in the real price of oil exhibits significant non-diagonality and asymmetry. I present evidence that increased uncertainty about the change in the real price of oil is associated with a lower average growth rate of real economic activity. Generalized impulse response experiments highlight the asymmetric effects of positive and negative shocks in the change in the real price of oil to output growth. Also, volatility impulse response experiments reveal that the effect of good news (negative shocks to the change in the real price of oil) on the conditional variance of the change in the real price of oil differs in magnitude and persistence from that of bad news of similar magnitude. This helps further in explaining the asymmetric response of output growth to oil price change shocks.

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Table 1. Summary Statistics

A. Summary Statistics

Variable	Mean	Variance	Skewness	Excess kurtosis	J-B normality
$\Delta \ln y_t$	2.682	74.062	−0.764	3.533	301.393
					(0.000)
$\Delta \ln oil_t$	3.558	8267.555	2.775	32.999	22768.284
					(0.000)

B. Unit Root and Stationary Tests

	Unit root tests			KPSS stationarity tests	
Variable	ADF(τ)	ADF(μ)	ADF	$\widehat{\eta}_\mu$	$\widehat{\eta}_\tau$
$\Delta \ln y_t$	−5.487	−6.321	−6.317	0.118	0.110
$\Delta \ln oil_t$	−7.309	−7.301	−7.525	0.273	0.020
5% cv	−1.941	−2.871	−3.425	0.463	0.146

C. Tests for Serial Correlation and ARCH

Variable	Q(4)	Q(12)	ARCH(4)
$\Delta \ln y_t$	63.838	80.037	12.615
	(0.000)	(0.000)	(0.013)
$\Delta \ln oil_t$	23.047	31.690	13.244
	(0.000)	(0.001)	(0.010)

D. Engle and Ng (1993) Tests for Sign and Size Bias in Variance

Variable	Sign	Negative size	Positive size	Joint
$\Delta \ln y_t$	24.440	−7.846	−1.622	37.102
	(0.013)	(0.000)	(0.171)	(0.000)
$\Delta \ln oil_t$	1679.703	−63.152	15.136	16.780
	(0.401)	(0.000)	(0.407)	(0.000)

Note: Numbers in parentheses are tail areas of tests. Annualized logarithmic first differences are used.

Figure 1. Logged Real Output and the Real Output Growth Rate

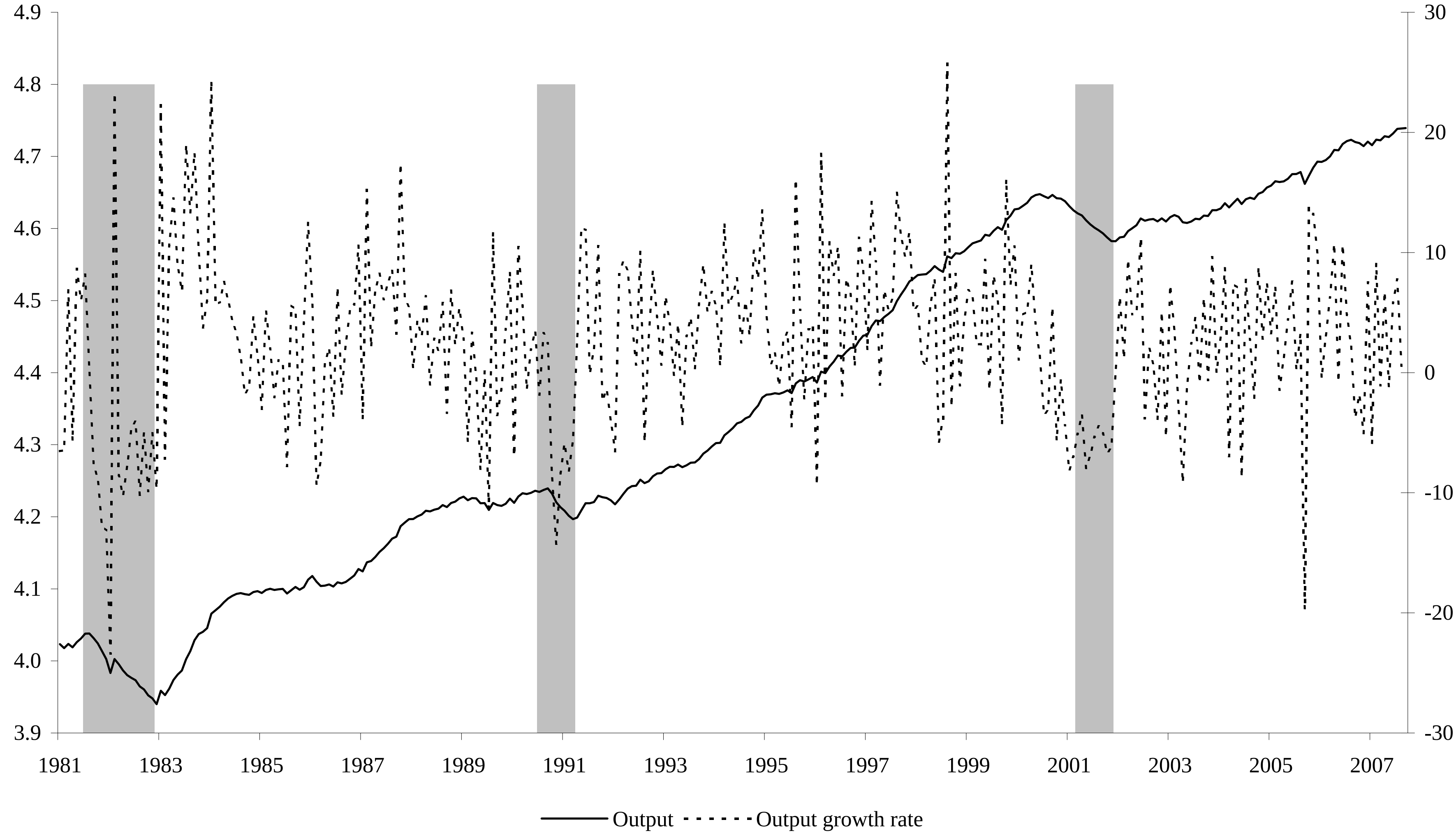

Figure 2. Logged Real Oil Price and the Rate of Change in the Real Price of Oil

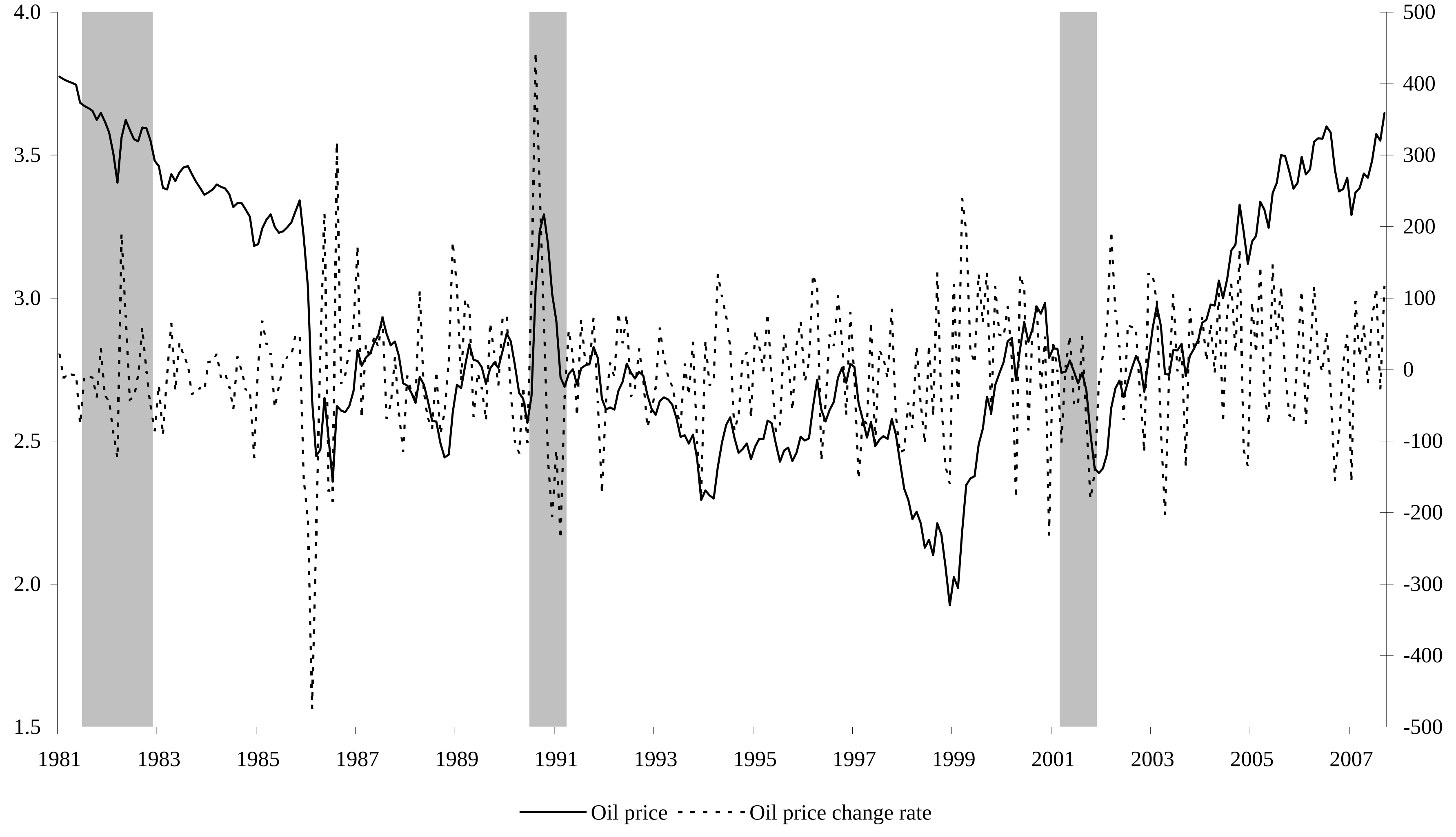

Table 2 The Bivariate Garch-In-Mean Asymmetric BEKK Model

Model: Equations (2) and (3) with $p = 3$, $q = 1$, and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \begin{matrix} 0.219 \\ (0.368) \end{matrix} \\ \begin{matrix} -64.560 \\ (0.000) \end{matrix} \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} \begin{matrix} 0.187 \\ (0.000) \end{matrix} & \begin{matrix} 0.005 \\ (0.141) \end{matrix} \\ \begin{matrix} 1.619 \\ (0.000) \end{matrix} & \begin{matrix} 0.180 \\ (0.002) \end{matrix} \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} \begin{matrix} 0.219 \\ (0.000) \end{matrix} & \begin{matrix} -0.006 \\ (0.051) \end{matrix} \\ \begin{matrix} 1.176 \\ (0.040) \end{matrix} & \begin{matrix} 0.159 \\ (0.004) \end{matrix} \end{bmatrix}; \mathbf{\Gamma}_3 = \begin{bmatrix} \begin{matrix} 0.170 \\ (0.000) \end{matrix} & \begin{matrix} -0.006 \\ (0.053) \end{matrix} \\ \begin{matrix} -0.558 \\ (0.337) \end{matrix} & \begin{matrix} 0.075 \\ (0.103) \end{matrix} \end{bmatrix};$$

$$\mathbf{\Psi}_0 = \begin{bmatrix} \begin{matrix} 1.606 \\ (0.000) \end{matrix} & \begin{matrix} -0.067 \\ (0.000) \end{matrix} \\ \begin{matrix} 4.635 \\ (0.000) \end{matrix} & \begin{matrix} -0.047 \\ (0.344) \end{matrix} \end{bmatrix}; \quad \mathbf{\Psi}_1 = \begin{bmatrix} \begin{matrix} -1.100 \\ (0.000) \end{matrix} & \begin{matrix} 0.041 \\ (0.000) \end{matrix} \\ \begin{matrix} -0.956 \\ (0.108) \end{matrix} & \begin{matrix} 0.454 \\ (0.000) \end{matrix} \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	0.037 (0.489)	0.962 (0.980)	3.933 (0.415)	7.940 (0.093)	144.434 (0.000)	53.536 (0.000)
z_{oil_t}	0.032 (0.550)	0.965 (0.982)	5.601 (0.230)	6.087 (0.192)	10.784 (0.547)	8.789 (0.720)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \begin{matrix} 3.869 \\ (0.000) \end{matrix} & \begin{matrix} 3.421 \\ (0.604) \end{matrix} \\ & \begin{matrix} 31.218 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \begin{matrix} -0.050 \\ (0.609) \end{matrix} & \begin{matrix} -1.328 \\ (0.055) \end{matrix} \\ \begin{matrix} 0.006 \\ (0.277) \end{matrix} & \begin{matrix} 0.565 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \begin{matrix} -0.657 \\ (0.000) \end{matrix} & \begin{matrix} 1.650 \\ (0.117) \end{matrix} \\ \begin{matrix} 0.010 \\ (0.143) \end{matrix} & \begin{matrix} 0.737 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \begin{matrix} 0.610 \\ (0.000) \end{matrix} & \begin{matrix} -0.422 \\ (0.785) \end{matrix} \\ \begin{matrix} 0.010 \\ (0.147) \end{matrix} & \begin{matrix} 0.157 \\ (0.350) \end{matrix} \end{bmatrix}.$$

Hypothesis testing

Diagonal VAR	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = 0$, for $i = 1, 2, 3$	(0.000)
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	(0.000)
No GARCH-M	$H_0 : \psi_{ij}^{k} = 0$, for all i, j, k	(0.000)
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	(0.000)
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	(0.136)

Note: Numbers in parentheses are tail areas of tests.

Table 3

Diagnostic Tests Based On The News Impact Curve

	$\varepsilon_{y_t}^2 - h_{y_t}$	$\varepsilon_{y_t}, \varepsilon_{oil_t} - h_{yoil_t}$	$\varepsilon_{oil_t}^2 - h_{oil_t}$
$I(e_{y_{t-1}} < 0)$	0.337 [0.561]	0.240 [0.623]	0.359[0.548]
$I(e_{oil_{t-1}} < 0)$	0.059 [0.807]	0.009 [0.923]	0.875[0.349]
$I(e_{y_{t-1}} < 0,\, e_{oil_{t-1}} < 0)$	0.122 [0.726]	0.108 [0.741]	0.953[0.328]
$I(e_{y_{t-1}} > 0,\, e_{oil_{t-1}} < 0)$	0.416 [0.518]	0.204 [0.650]	4.433[0.035]
$I(e_{y_{t-1}} < 0,\, e_{oil_{t-1}} > 0)$	0.102 [0.748]	0.817 [0.365]	0.085[0.770]
$I(e_{y_{t-1}} > 0,\, e_{oil_{t-1}} > 0)$	0.001 [0.970]	0.982 [0.321]	1.810[0.178]
$e_{y_{t-1}}^2 I(e_{y_{t-1}} < 0)$	4.424 [0.035]	0.275 [0.599]	1.841[0.174]
$e_{y_{t-1}}^2 I(e_{oil_{t-1}} < 0)$	2.865 [0.090]	0.393 [0.530]	4.520[0.033]
$e_{oil_{t-1}}^2 I(e_{y_{t-1}} < 0)$	2.849 [0.091]	0.333 [0.563]	4.056[0.044]
$e_{oil_{t-1}}^2 I(e_{oil_{t-1}} < 0)$	2.556 [0.109]	0.080 [0.766]	0.220[0.638]

Note: Numbers are tail areas of tests.

Figure 3. Conditional Standard Deviation of Output Growth

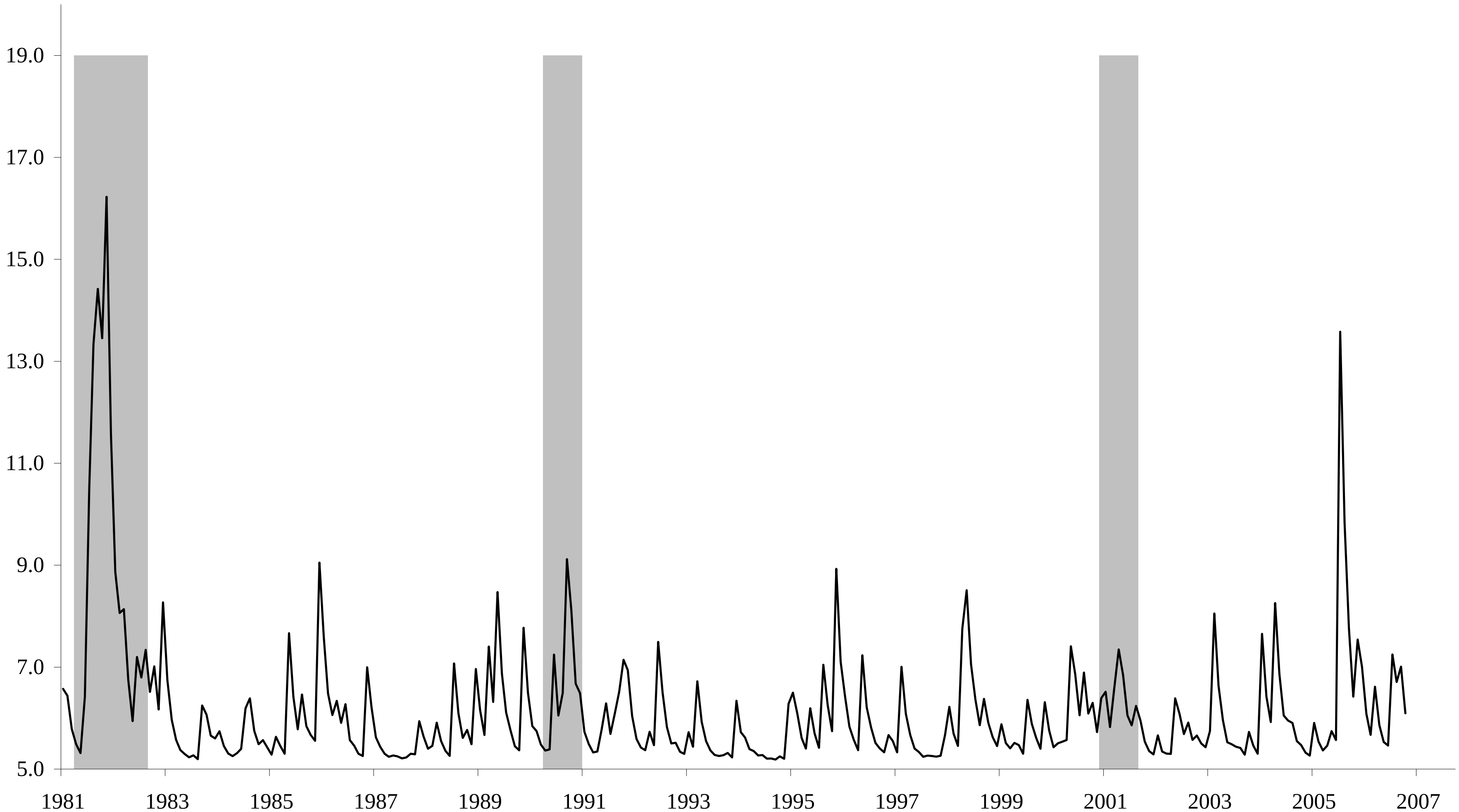

Figure 4. Conditional Standard Deviation of the Change in the Price of Oil

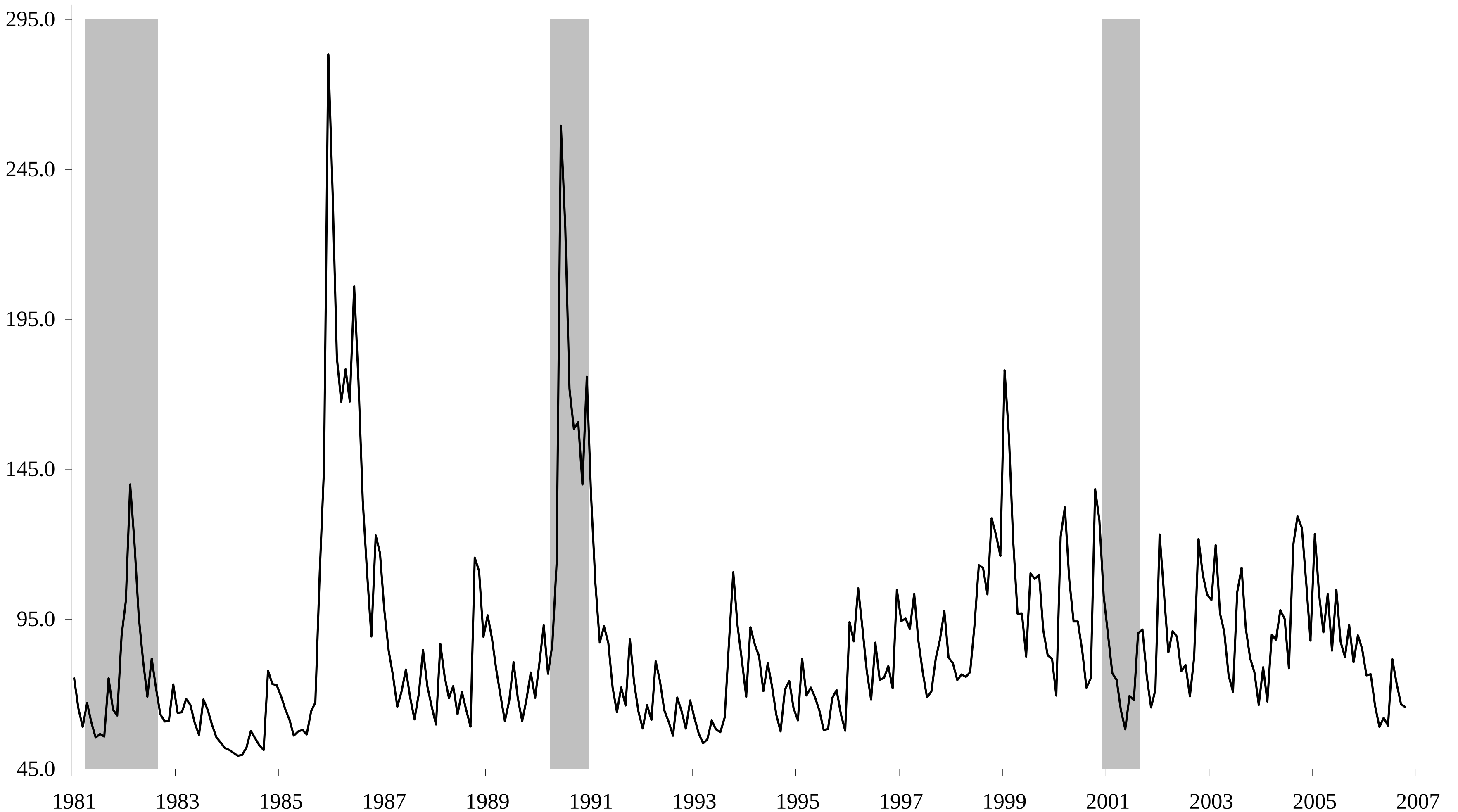

Figure 5. Covariance Between Output Growth and the Change in the Price of Oil

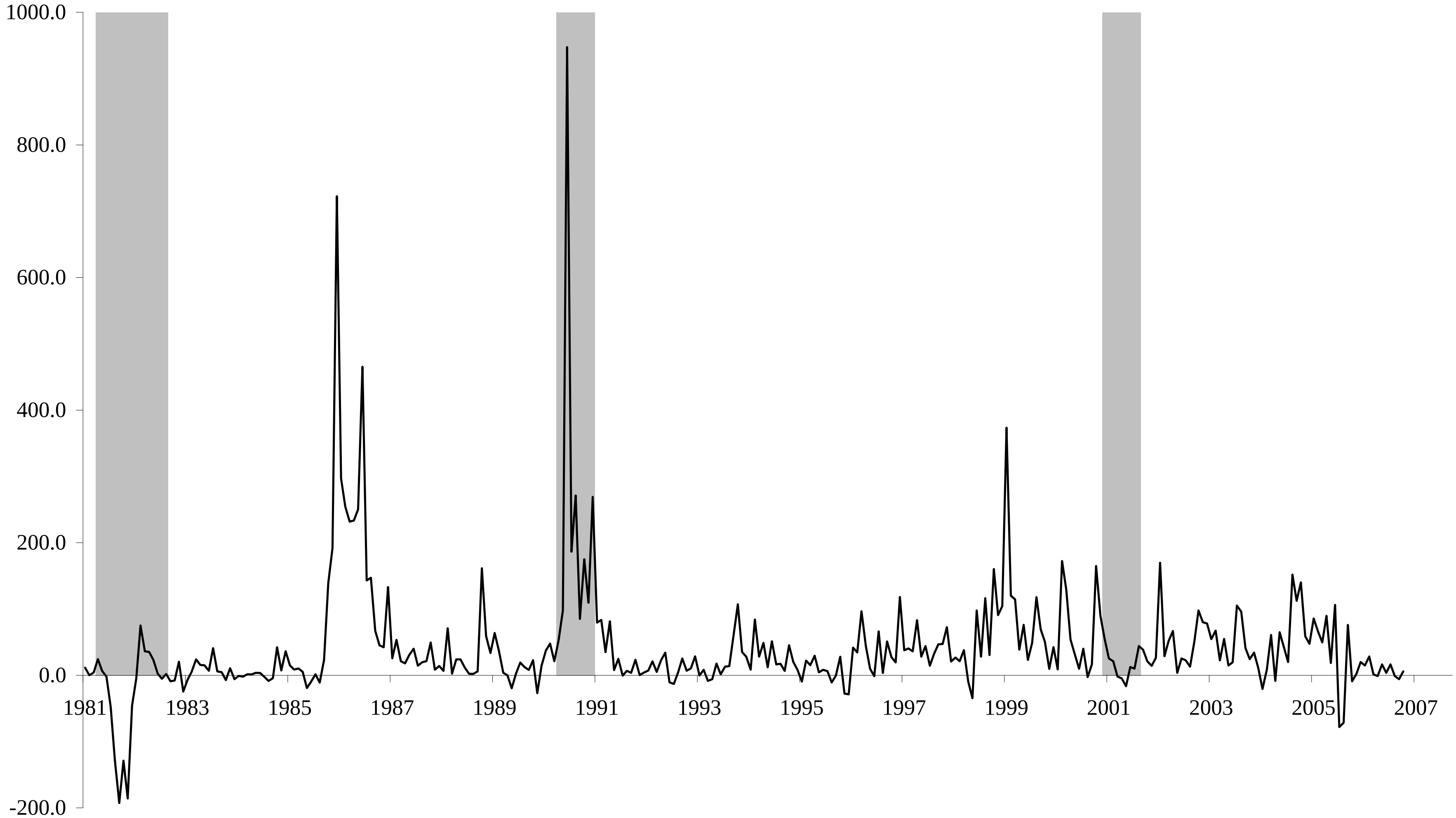

Figure 6. GIRF of Output Growth to an Oil Price Change Shock

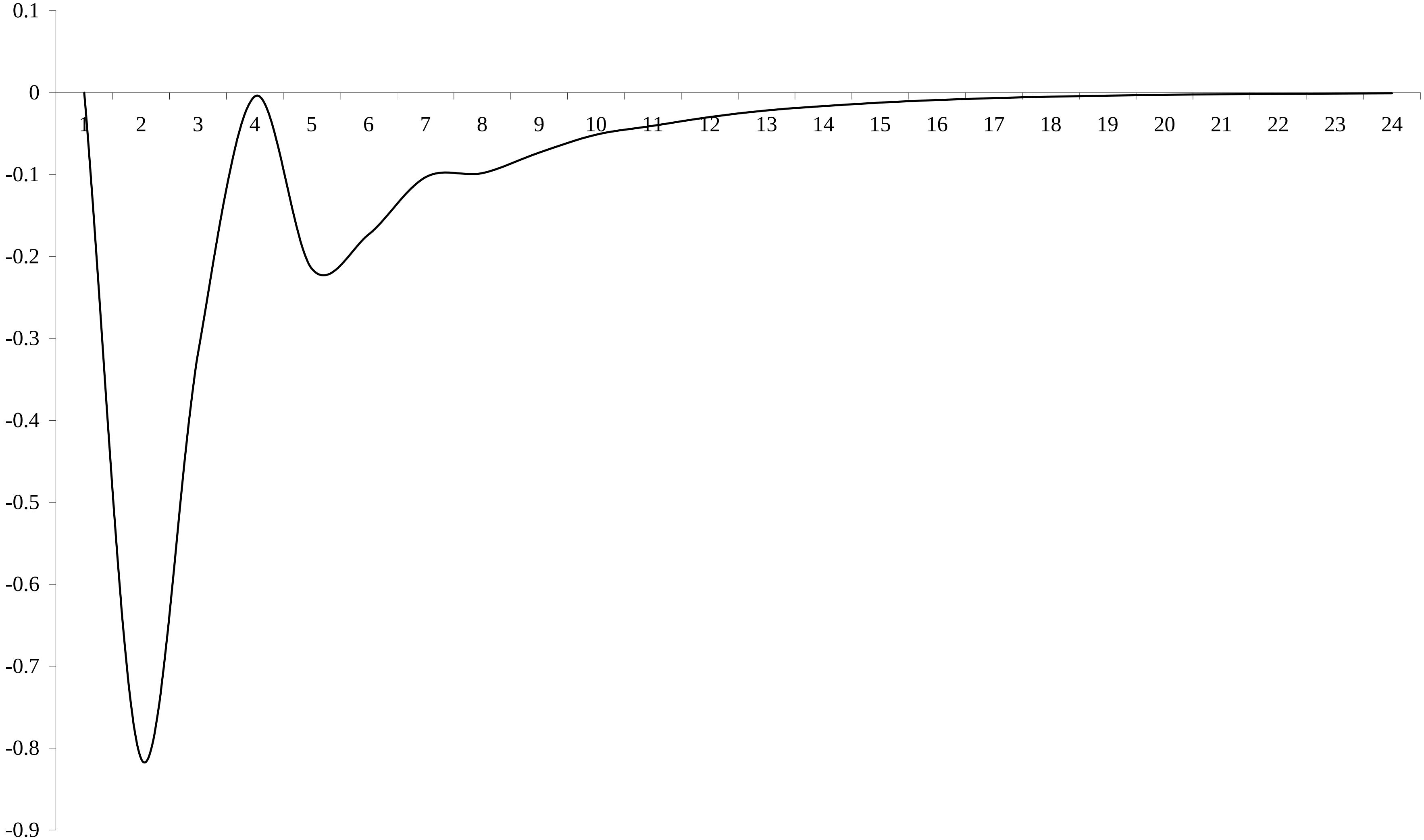

Figure 7. GIRF of the Oil Price Change to an Output Growth Shock

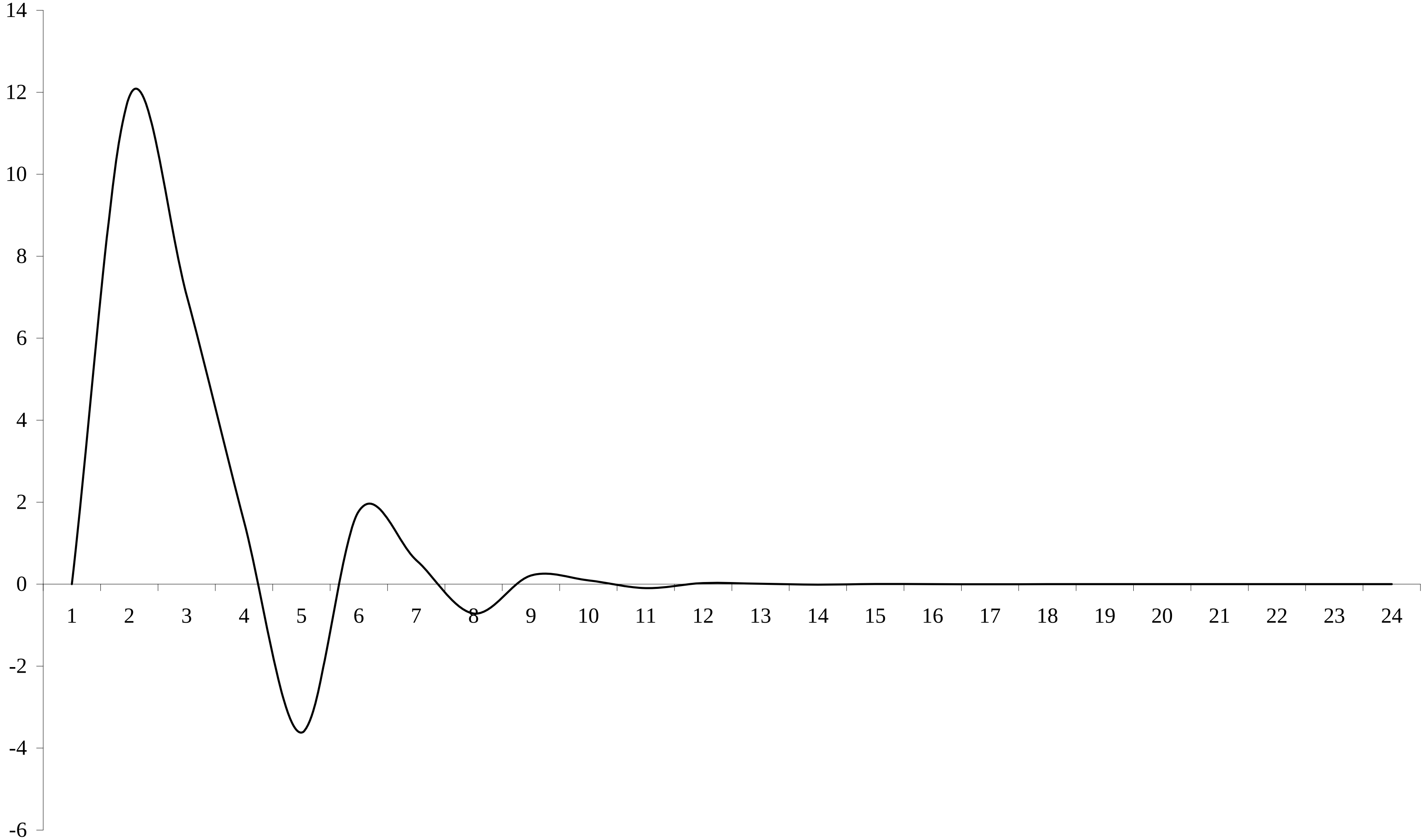

Figure 8. GIRFs of Output Growth to Positive and Negative Shocks to the Change in the Price of Oil

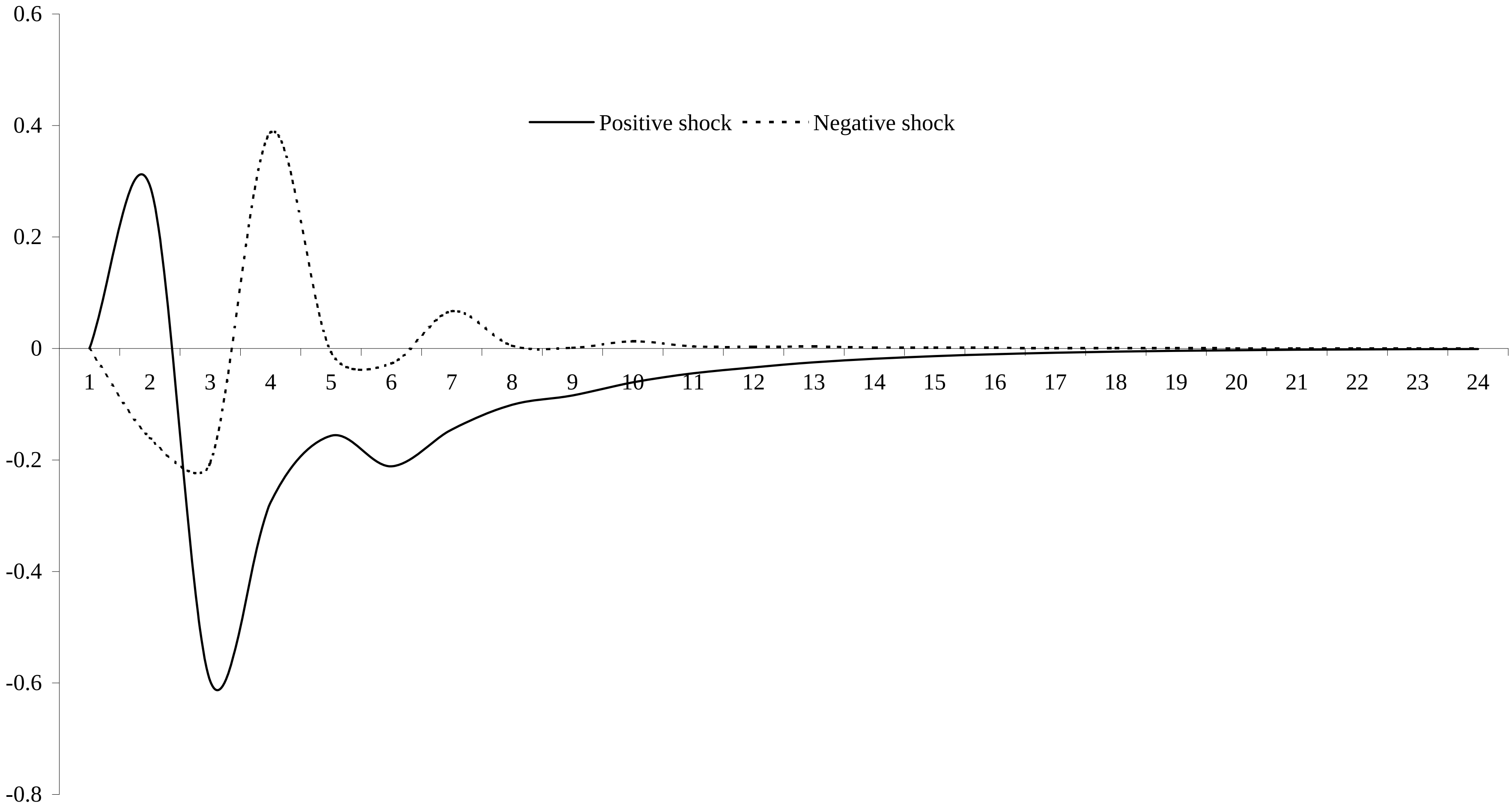

Figure 9. GIRFs of the Change in the Price of Oil to Positive and Negative Output Growth Shocks

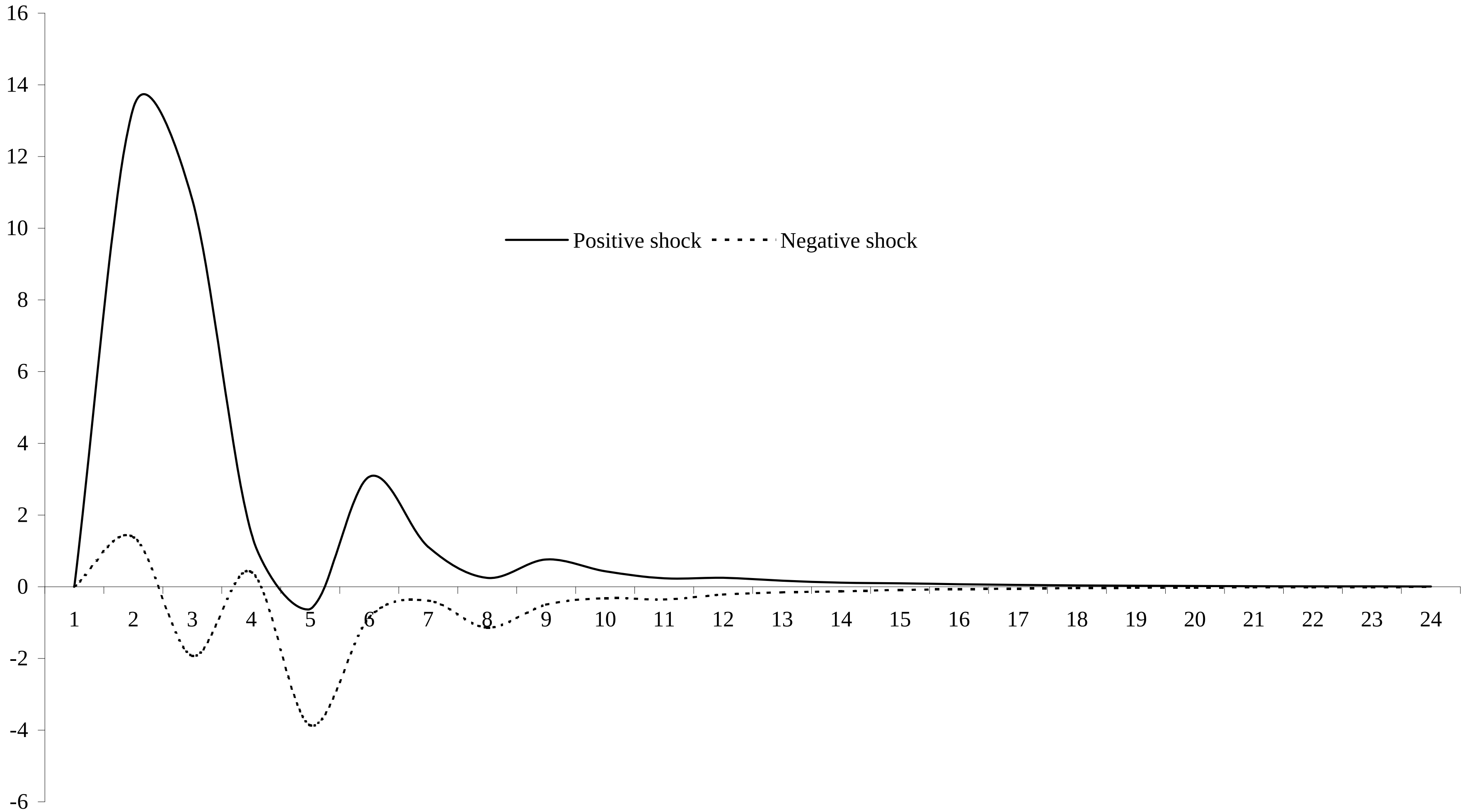

Figure 10. The Distribution of the Asymmetry Measure Based on the GIRFs of Output Growth to Shocks in the Change in the Price of Oil

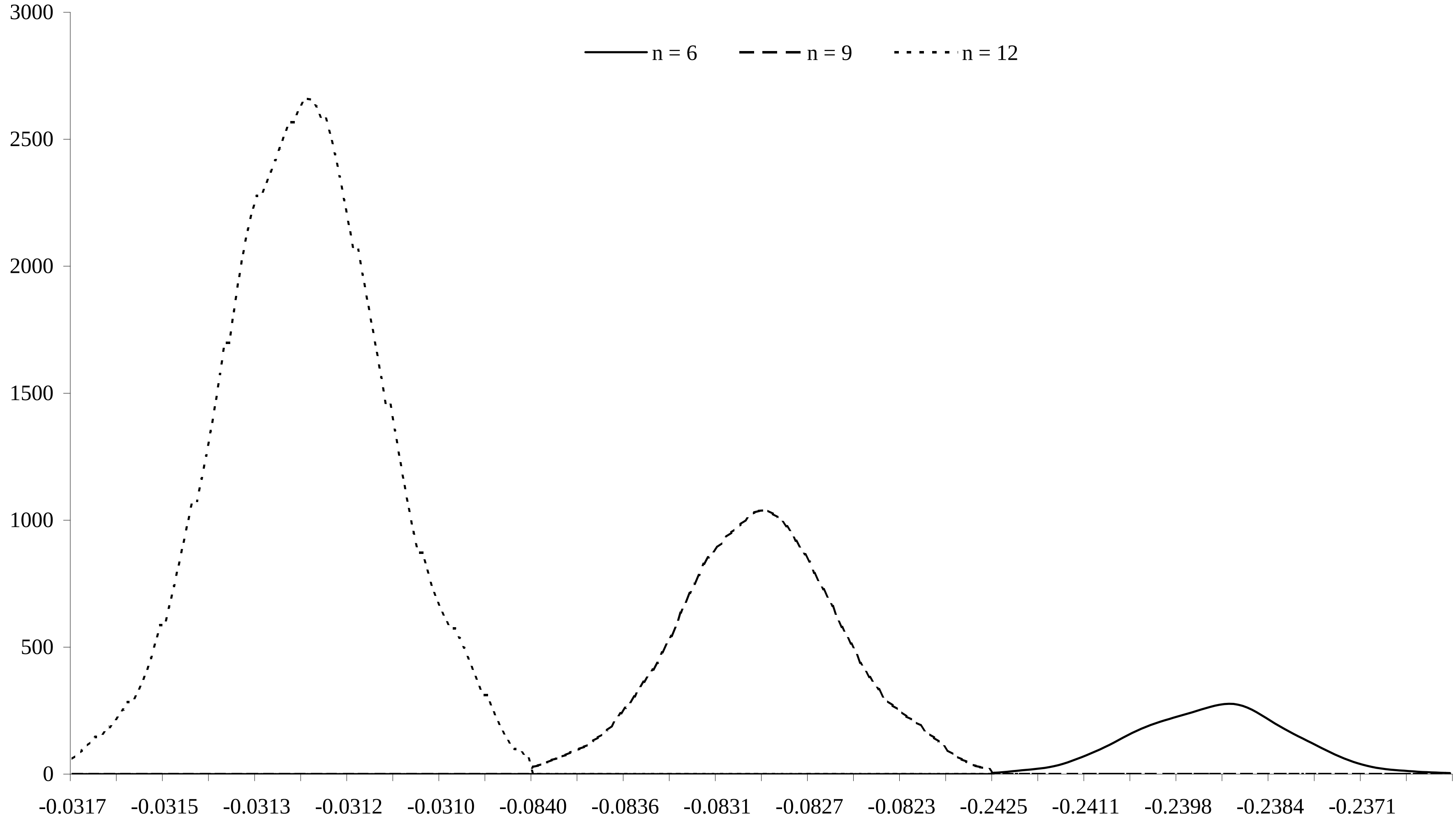

Figure 11. The Distribution of the Asymmetry Measure Based on the GIRFs of the Change in the Price of Oil to Shocks in Output Growth

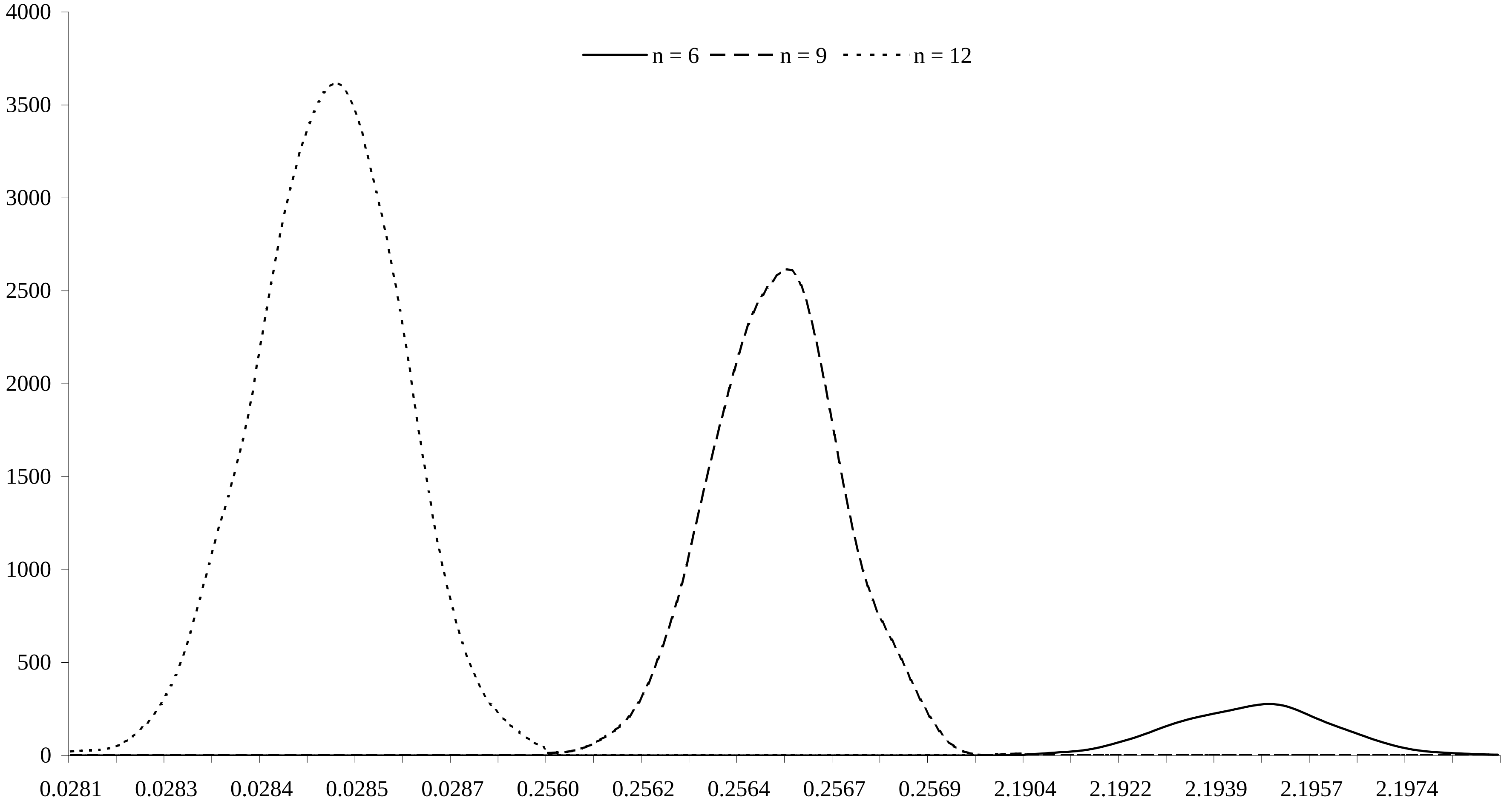

Figure 12. Volatility Responses to Oil Price Change Shocks

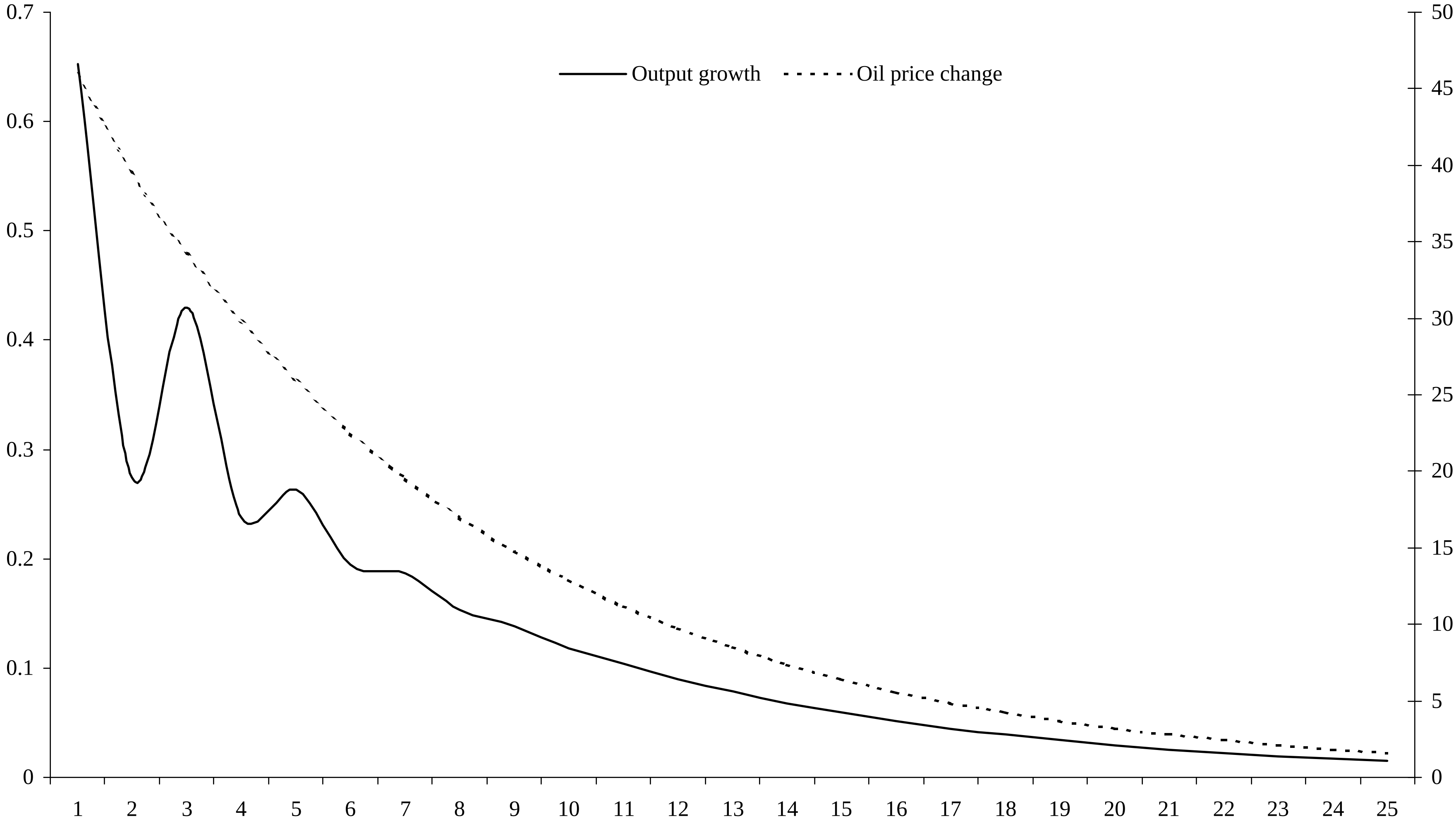

Figure 13. Volatility Responses to Output Growth Shocks

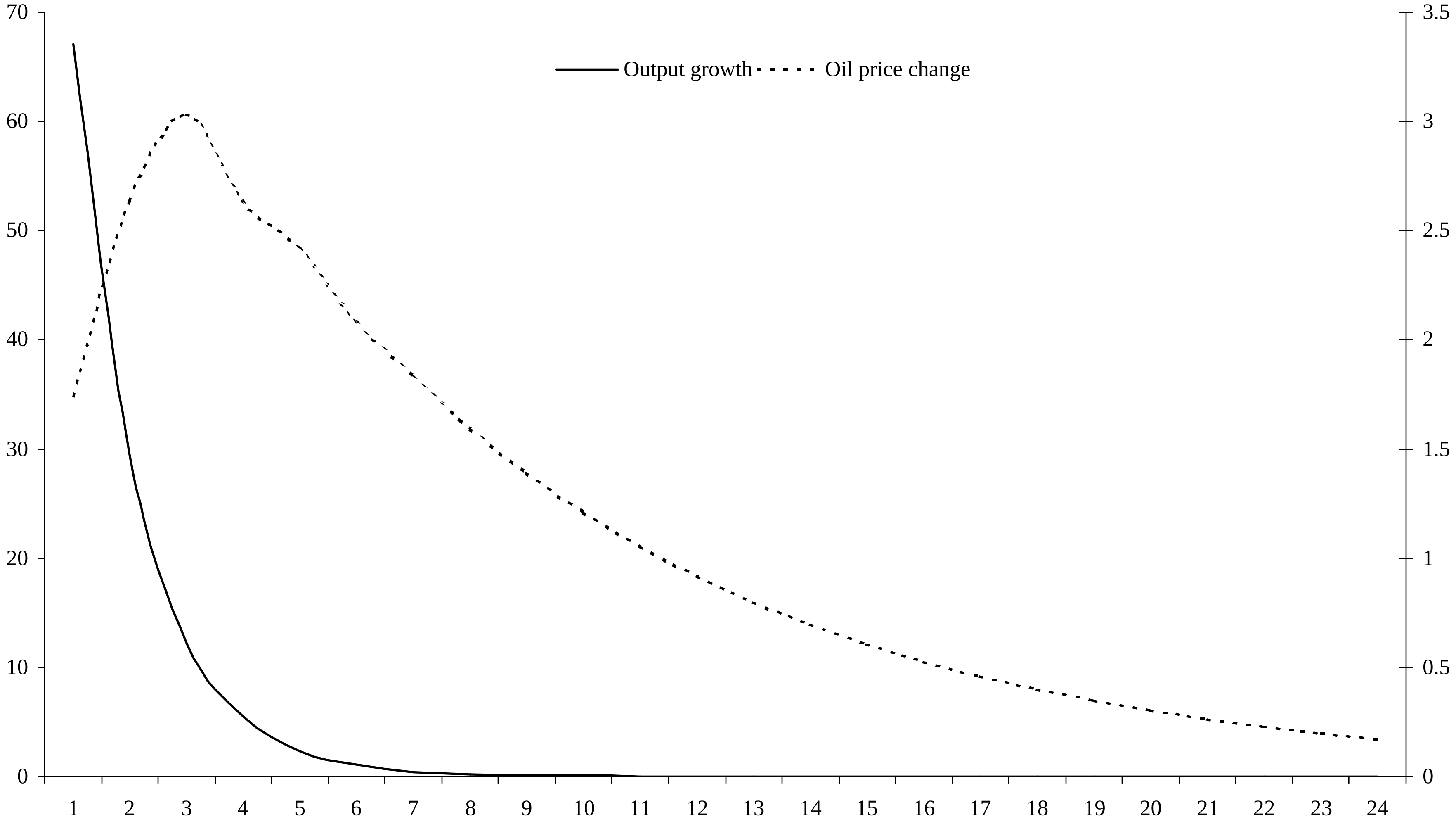

Figure 14. VIRFs of Oil Price Change to Positive and Negative Shocks to the Change in the Price of Oil

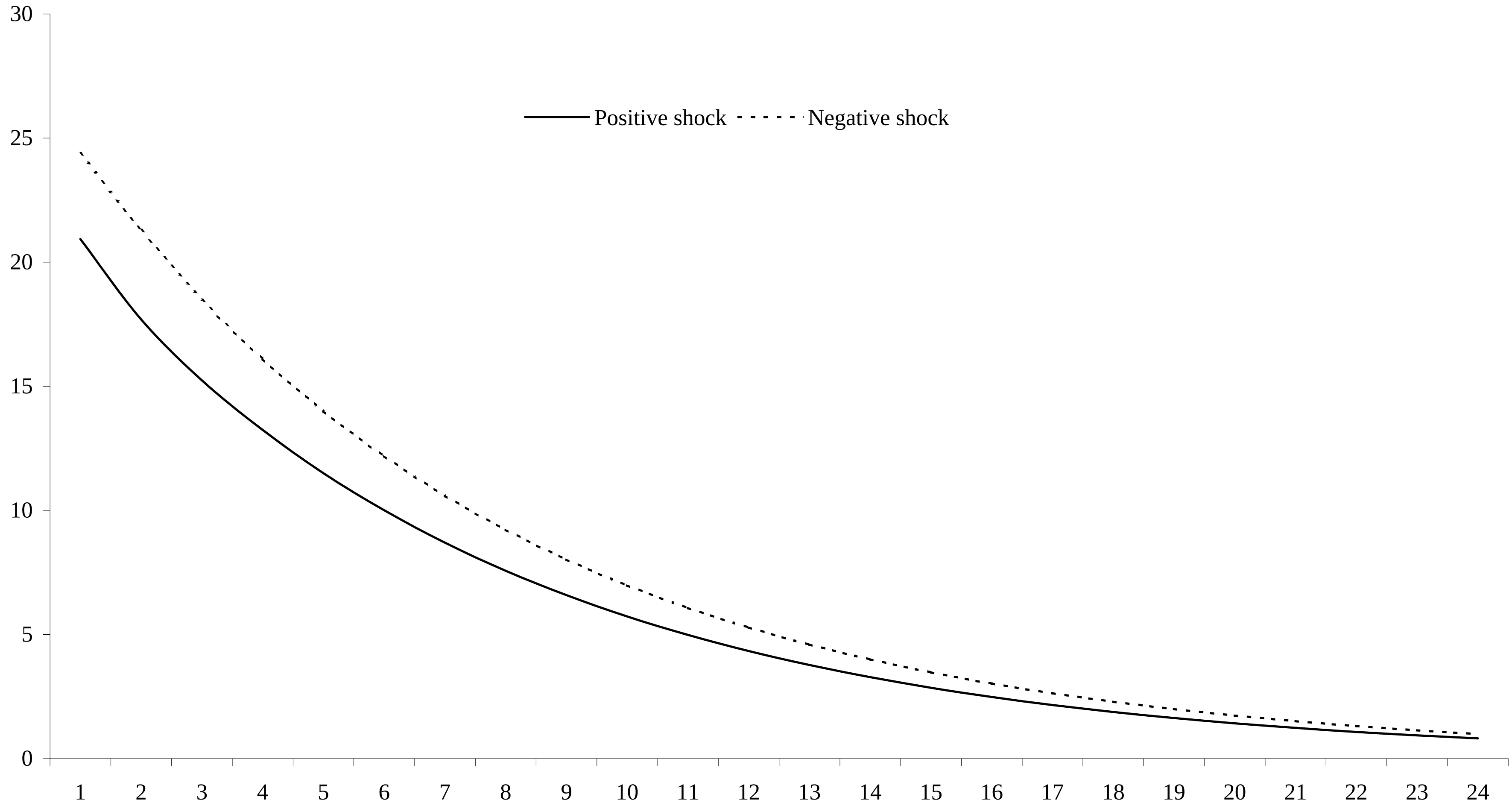

Figure 15. The Distribution of the Asymmetry Measure Based on the VIRFs of tl
Change in Price of oil to Shocks in the Change in the Price of Oil

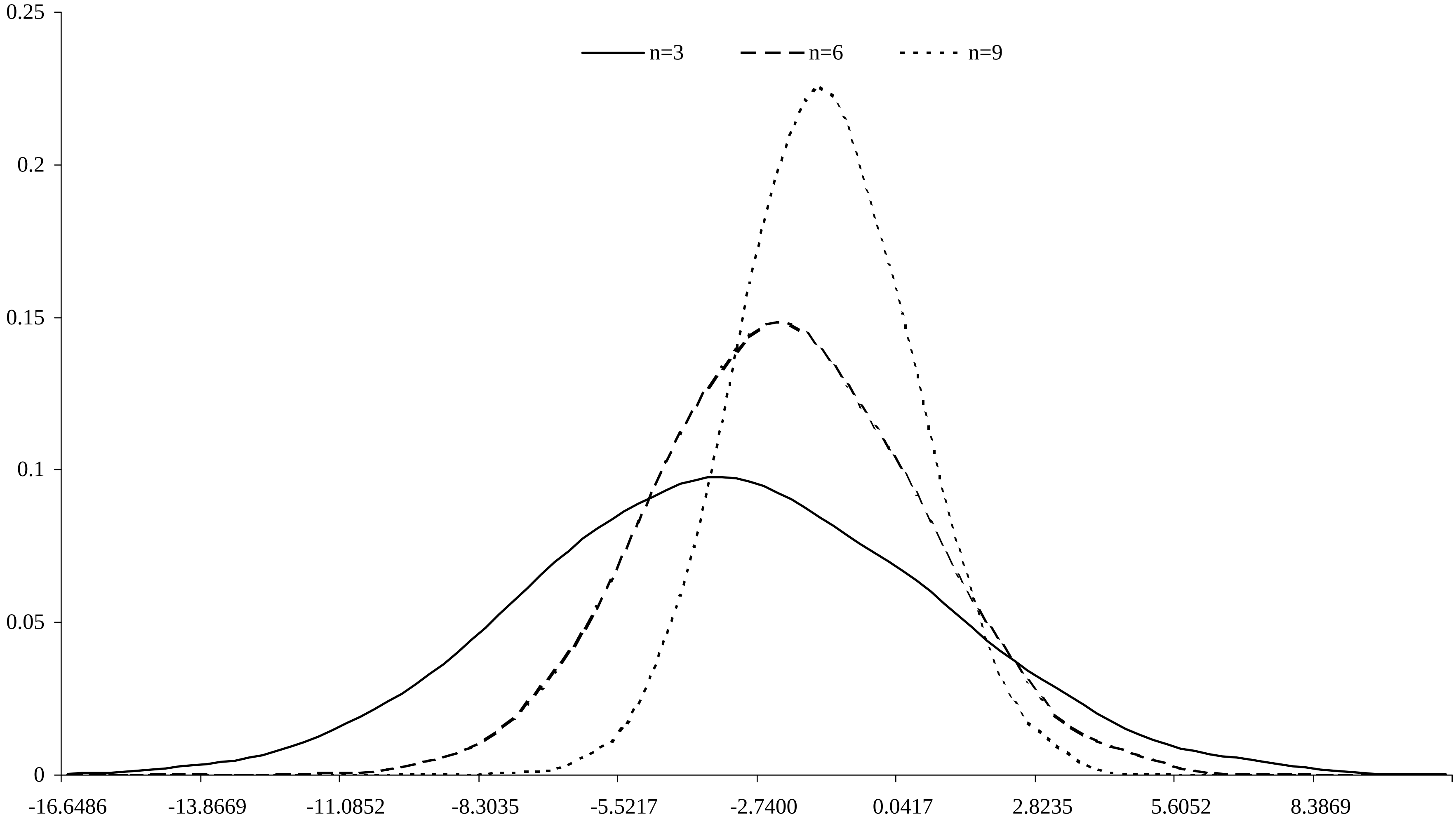

Figure 16. The Distribution of the Asymmetry Measure Based on the VIRFs of tl
Change in the Price of Oil to Shocks in Output Growth