Modelling Asymmetric Cointegration and Dynamic Multipliers in a Nonlinear ARDL Framework

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Abstract

This paper develops a cointegrating nonlinear ARDL (NARDL) model in which short- and long-run nonlinearities are introduced via positive and negative partial sum decompositions of the explanatory variables. We demonstrate that the model is estimable by OLS and that reliable long-run inference can be achieved by bounds-testing regardless of the integration orders of the variables. Furthermore, we derive asymmetric dynamic multipliers that graphically depict the traverse between the short- and the long-run. The salient features of the model are illustrated using the examples of nonlinearities in both the unemployment-output relationship and the adjustment of retail gasoline prices.

JEL Classification: C12, C13, J64.

Key Words: Asymmetric Cointegrating Relationships, Asymmetric Dynamic Multipliers, Nonlinear ARDL (NARDL) ECM-based Estimation and Tests, Nonlinear Unemployment-Output Relationship, Asymmetric Gasoline Price Adjustment.

1 Introduction

The nonlinearity of many macroeconomic variables and processes has long been recognised. In a famous remark, Keynes (1936, p. 314) noted that "the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning point when an upward is substituted for a downward tendency". More recently, the joint fields of behavioural finance and economics associated most notably with Daniel Kahneman, Amos Tversky and Robert Shiller (*e.g.* Kahneman and Tversky, 1979; Shiller, 1993, 2005) have provided a considerable impetus to the modelling of asymmetry, stressing that nonlinearity is endemic within the social sciences and that asymmetry is fundamental to the human condition.

Since the mid-nineties, a substantial literature has considered the joint issues of nonstationarity and nonlinearity. This field has been dominated by three regime-switching models: the threshold ECM associated with Balke and Fomby (1997), the Markov-switching ECM of Psaradakis *et al.* (2004), and the smooth transition regression ECM developed by Kapetanios *et al.* (2006). The development of this literature reflects the belief that the information revealed by linear models may be insufficiently rich to permit strong inference or to yield reliable forecasts. More generally, it suggests a general concern that the assumption of linear adjustment may be excessively restrictive in a wide range of economically interesting situations, particularly where transaction costs are non-negligible and where policy interventions are observed in-sample.

The majority of these studies, however, maintain the assumption that the long-run relationship may be represented as a symmetric linear combination of nonstationary stochastic regressors. With the notable exceptions of Park and Phillips (2001), Saikkonen and

Choi (2004), Escribano *et al.* (2006) and Bae and de Jong (2007), little research effort has been devoted to the analysis of nonlinear cointegration. Schorderet (2001, 2003) has proposed the bivariate asymmetric cointegrating regression of unemployment on output, where output is decomposed into partial sum processes of positive and negative changes. On the basis of this piecewise linear specification, he finds that the impact of recessions on unemployment is larger in absolute terms than that of cyclical upturns, indicating an hysteretic relationship. Granger and Yoon (2002) further develop the notion that the cointegrating relationship may be defined between the positive and negative components of the underlying variables, an effect that they term 'hidden cointegration'.

Partial sum decompositions have been applied with some success to the analysis of dynamic asymmetry. Examples include Webber's (2000) analysis of the relationship between the exchange rate and import prices, Lee (2000) and Virén's (2001) work on asymmetries in Okun's Law and the research of Borenstein *et al.* (1997) and Bachmeier and Griffin (2003) focusing on the asymmetric response of gasoline prices to fluctuations in the oil price. However, most papers modelling short-run asymmetry employ the two step Engle-Granger technique which is inherently less efficient than single-step ECM estimation. Moreover, papers coherently modelling long- and short-run asymmetries jointly are scarce.

Our purpose in this paper is to develop a simple and flexible nonlinear dynamic framework capable of simultaneously and coherently modelling asymmetries both in the underlying long-run relationship and in the patterns of dynamic adjustment. We make four principle contributions. Firstly, we derive the dynamic error correction representation associated with the asymmetric long-run cointegrating regression, resulting in the nonlinear

ARDL (NARDL) model. Secondly, following Pesaran and Shin (1998) and Pesaran *et al.* (2001), we employ a pragmatic bounds-testing procedure for the existence of a stable long-run relationship which is valid irrespective of whether the underlying regressors are $I(0)$, $I(1)$ or mutually cointegrated. Thirdly, we derive asymmetric cumulative dynamic multipliers that allow us to trace out the asymmetric adjustment patterns following positive and negative shocks to the explanatory variables. This has substantial theoretical appeal as it allows us to depict in an intuitive manner the traverse to a new equilibrium following a perturbation to the system. Such is the flexibility of our framework that it can readily accommodate the four general combinations of long- and short-run asymmetry. Finally, we conduct a range of Monte Carlo experiments which largely validate our estimation and inferential framework, revealing little bias in estimation and considerable power of the key test statistics. Moreover, we compute empirical p-values for the cointegration tests and confidence intervals for our dynamic multipliers by means of a non-parametric bootstrap. These exercises highlight a further enviable attribute of our proposed methodology: it is easily estimable by OLS and simple inferential methods provide a straightforward and reliable means of discriminating between the various forms and combinations of asymmetries.

We demonstrate the usefulness of the NARDL framework through two empirical applications. First, we investigate the unemployment-output relationship in the US, Canada and Japan over the period 1982m2–2003m11. We find strong evidence of long-run asymmetry consistent with the growing consensus that unemployment is more sensitive to busts than booms. Moreover, particularly in Canada, we find dynamic asymmetries indicating that firms are quick to fire and slow to hire. Finally, the dynamic multipliers reveal a pattern that is often obscured in discussions of persistence – although the half-life of an expansionary shock in the US is smaller than that of an equivalent recessionary shock, the real impact in terms of jobs created/lost is larger in the recessionary case. It follows, therefore, that focusing on the half-life of a shock is insufficient when the long-run relationship is asymmetric as this fails to convey relevant information about the relative magnitude of the economic response to the shock in each regime.

Our second application investigates the asymmetric responses of Korean retail gasoline prices to fluctuations in the crude oil spot price and the Korean Won/US Dollar exchange rate over the period 1991q1–2007q2. Our results indicate that the long-run relationship is linear in both variables, indicating that retailers pass cost changes through to consumers symmetrically in the long-run. However, the speed of upward adjustment exceeds that of downward adjustment, indicating a degree of downward price-stickiness consistent with the 'rockets and feathers' hypothesis associated with Bacon (1991). Moreover, our results support the findings of Asplund *et al.* (2000) that the short-run response of gasoline prices to the exchange rate is more pronounced than that associated with fluctuations in the price of crude oil.

Finally, the flexibility and utility of the NARDL technique is reflected in the growing literature that has adopted our technique for the analysis of a range of economic issues[1]. Van Treeck (2008) has employed the NARDL model in his analysis of asymmetric wealth effects on US consumption, and has found that liquidity constraints and loss-aversion can be reconciled inter-temporally, with the former dominating in the short-run and the latter in the long-run. More recently, Delatte and López-Villavicencio (2010, 2011) have applied the NARDL technique in their analysis of long-run asymmetries in the pass-through from exchange rates to consumer prices in developed economies. Nguyen and Shin (2010) have estimated NARDL models on high frequency exchange rate data, revealing interesting patterns of asymmetry in the pricing impacts of order flow. Lastly, Greenwood-Nimmo, Shin and Van Treeck (2011) have estimated NARDL models of the interest rate pass-through relationship in the USA and Germany, finding strong evidence of time-varying asymmetry. An important and relatively common finding in this literature is that the direction of asymmetry may switch between the short-run and the long-run. For example, a positive shock may have a larger absolute effect in the short-run while a negative shock has a larger absolute effect in the long-run (or *vice-versa*). The simplicity and flexibility of NARDL renders it an ideal framework with which to model such complex phenomena.

The paper proceeds as follows. Section 2 introduces the asymmetric cointegrating regression model and derives the associated asymptotic theory. On this basis, the NARDL model is derived including expressions for the asymmetric cumulative dynamic multipliers, and the associated testing procedures are developed. Section 3 employs a range of Monte Carlo simulations to investigate the finite sample properties of the proposed estimators and the test statistics. Section 4 presents the results of our two empirical illustrations. Lastly, Section 5 offers some concluding remarks, while mathematical proofs are collected in the Appendix.

2 Modelling Asymmetries in a Nonlinear ARDL Framework

The increasing popularity of nonlinear modelling in the context of cointegrating long-run relationships has led to the proliferation of regime-switching models. Among existing studies, nonlinearity is typically confined to the error correction mechanism and estimation proceeds on the basis of either the threshold ECM associated with Balke and Fomby (1997), the Markov-Switching ECM of Psaradakis *et al.* (2004) or the smooth transition regression ECM developed by Kapetanios *et al.* (2006). However, the common assumption that the underlying cointegrating relationship may be represented as a linear combination of the underlying nonstationary variables may be excessively restrictive. In general, the long-run (cointegrating) relationship may also be subject to asymmetry or nonlinearity.[2] The three regime-switching type functional forms mentioned above are equally applicable to the case of long-run asymmetry (Saikkonen and Choi, 2004; Escribano *et al.*, 2006).

In principle, it is possible to obtain a unified model capable of combining nonlinearities in the long-run relationship and the error correction mechanism coherently. In practice, however, selection of the regime-switching variables and the transition functional forms may be non-trivial.[3] Hence, the development of an operational model of this form is likely to be highly challenging (c.f. Saikkonen, 2008). We contribute to this literature by developing a nonlinear modelling framework based on the ARDL approach which provides a simple and flexible vehicle for the analysis of joint long- and short-run asymmetries.

2.1 Nonlinear Asymmetric Cointegration

Before developing the full representation of the NARDL model, we introduce the following asymmetric long-run regression:

$$y_t = \beta^+ x_t^+ + \beta^- x_t^- + u_t, \tag{2.1}$$

$$\Delta x_t = v_t, \tag{2.2}$$

where y_t and x_t are scalar I(1) variables, and x_t is decomposed as $x_t = x_0 + x_t^+ + x_t^-$ where x_t^+ and x_t^- are partial sum processes of positive and negative changes in x_t:

$$\boldsymbol{x}_t^+ = \sum_{j=1}^{t} \Delta \boldsymbol{x}_j^+ = \sum_{j=1}^{t} \max\left(\Delta \boldsymbol{x}_j, 0\right), \ \boldsymbol{x}_t^- = \sum_{j=1}^{t} \boldsymbol{\Delta} x_j^- = \sum_{j=1}^{t} \min\left(\Delta \boldsymbol{x}_j, 0\right). \tag{2.3}$$

This simple approach to modelling asymmetric cointegration based on partial sum decompositions has been applied by Schorderet (2001) in the context of the nonlinear relationship between unemployment and output.[4]

Granger and Yoon (2002) advance the concept of 'hidden cointegration', where cointegrating relationships may be defined between the positive and negative components of the underlying variables. They demonstrate the relevance of this conceptual framework in the context of the linkage between US short- and long-term interest rates and the output-unemployment relationship, both of which are notable for the lack of robust evidence of linear cointegration. Schorderet (2003) generalises this concept and defines the following stationary linear combination of the partial sum components:

$$z_t = \beta_0^+ y_t^+ + \beta_0^- y_t^- + \beta_1^+ x_t^+ + \beta_1^- x_t^-. \tag{2.4}$$

If z_t is stationary, then y_t and x_t are said to be 'asymmetrically cointegrated'. It follows that standard linear (symmetric) cointegration is a special case of (2.4), obtained only if $\beta_0^+ = \beta_0^-$ and $\beta_1^+ = \beta_1^-$. Schorderet modifies (2.4) to analyse hidden cointegration, where only one component of each series appears in (2.4), developing a model of the asymmetric cointegrating relationship between bilateral exchange rates as an illustration. Lardic and Mignon (2008) analyse hidden cointegration between the price of oil and GDP, although they fail to provide any economically meaningful interpretation of the estimated asymmetric coefficients.

Given the difficulty in interpreting the results of hidden cointegration analysis, we will focus on (2.1), imposing the restriction $\beta_0^+ = \beta_0^- = \beta_0$ in (2.4) such that $\beta^+ = -\beta_1^+/\beta_0$ and $\beta^- = -\beta_1^-/\beta_0$. To achieve the greatest possible clarity of exposition, we initially begin with the case of a single regressor decomposed into the relevant partial sum processes.

Assumption 1 *The disturbances u_t and v_t in (2.1) and (2.2) follow iid processes with zero means and finite variances, and they are independently distributed.*

Theorem 1 *Consider the asymmetric cointegrating regression, (2.1) and (2.2). Under Assumption 1, the OLS estimators of β^+ and β^- have the following asymptotic distributions:*

$$T(\hat{\beta}^+ - \beta^+) \Rightarrow -\left(\frac{\mu^-\sigma^u}{\sigma^s}\right)\frac{\frac{1}{3}\int W_{\tilde{s}}(r)dW_{\tilde{u}}(r) - \int rW_{\tilde{s}}(r)dr\left(W_{\tilde{u}}(1) - \int W_{\tilde{u}}(r)dr\right)}{\frac{1}{3}\int W_{\tilde{s}}(r)^2dr - \left(\int rW_{\tilde{s}}(r)dr\right)^2},$$

$$T(\hat{\beta}^- - \beta^-) \Rightarrow \left(\frac{\mu^+\sigma^u}{\sigma^s}\right)\frac{\frac{1}{3}\int W_{\tilde{s}}(r)dW_{\tilde{u}}(r) - \int rW_{\tilde{s}}(r)dr\left(W_{\tilde{u}}(1) - \int W_{\tilde{u}}(r)dr\right)}{\frac{1}{3}\int W_{\tilde{s}}(r)^2dr - \left(\int rW_{\tilde{s}}(r)dr\right)^2},$$

where $\mu^+ := E[\max[0, v_t]]$, $\mu^- := E[\min[0, v_t]]$, $s_t := \mu^+ \min[0, v_t] - \mu^- \max[0, v_t]$, $\sigma_u^2 := Var(u_t)$, $\sigma_s^2 := Var(s_t)$, and $W_{\tilde{s}}(\cdot)$ and $W_{\tilde{u}}(\cdot)$ are two independent standard Brownian motions defined on $r \in [0, 1]$, and obtained as the weak limit of partial sum processes,

$T^{-1/2}\sum_{j=1}^{T(\cdot)}\tilde{s}_t$ *and* $T^{-1/2}\sum_{j=1}^{T(\cdot)}\tilde{u}_t$*, with* $\tilde{u}_t := u_t/\sigma_u$ *and* $\tilde{s}_t := s_t/\sigma_s$*. Furthermore,*

$$T\{\mu^+(\hat{\beta}^+ - \beta^+) + \mu^-(\hat{\beta}^- - \beta^-)\} = o_p(1).$$

Remark 1 *In the special case when* v_t *follows a symmetric distribution with* ${\mu^+}^2 = {\mu^-}^2$ *and* $Var\left(\max[0, v_t]\right) = Var\left(\min[0, v_t]\right)$*,*[5] *then we have*

$$T(\hat{\beta}^- - \beta^-),\ T(\hat{\beta}^+ - \beta^+) \Rightarrow \frac{\frac{1}{3}\int W_{\tilde{s}}(r)dW_{\tilde{u}}(r) - \int rW_{\tilde{s}}(r)dr\left(W_{\tilde{u}}(1) - \int W_{\tilde{u}}(r)dr\right)}{\frac{1}{3}\int W_{\tilde{s}}(r)^2dr - \left(\int rW_{\tilde{s}}(r)dr\right)^2},$$

such that $T\left\{(\hat{\beta}^- - \beta^-) + (\hat{\beta}^+ - \beta^+)\right\} = o_p(1)$*.*

Remark 2 *Let* $\boldsymbol{\beta} = (\beta^+, \beta^-)'$*, then*

$$T\left(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}\right) \overset{a}{\sim} MN\left(0, \boldsymbol{V}\right), \tag{2.5}$$

where $\boldsymbol{V} = plim_{T\to\infty}T^2\left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\sigma_u^2$*. Even though* x_t^+ *and* x_t^- *are dominated by the deterministic trends by construction, these leading terms cancel off in the derivation of* $\left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}$ *such that* $plim_{T\to\infty}T^2\left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}$ *is well-defined and standard inference on* $\boldsymbol{\beta}$ *remains asymptotically valid.*

Remark 3 *In a similar manner, when an intercept term is included, we can obtain the asymptotic distributions of the OLS estimator as follows:*

$$T(\hat{\beta}^+ - \beta^+) \Rightarrow -\left(\frac{\mu^-\sigma^u}{\sigma^s}\right)\frac{\frac{1}{12}\int \tilde{W}_{\tilde{s}}(r)dW_{\tilde{u}}(r) - \left(\int(r-\frac{1}{2})\tilde{W}_{\tilde{s}}(r)dr\right)\left(\int(r-\frac{1}{2})dW_{\tilde{u}}(r)\right)}{\frac{1}{12}\int \tilde{W}_{\tilde{s}}(r)^2dr - \left(\int(r-\frac{1}{2}\tilde{W}_{\tilde{s}}(r)\right)^2};$$

$$T(\hat{\beta}^- - \beta^-) \Rightarrow \left(\frac{\mu^+\sigma^u}{\sigma^s}\right)\frac{\frac{1}{12}\int \tilde{W}_{\tilde{s}}(r)dW_{\tilde{u}}(r) - \left(\int(r-\frac{1}{2})\tilde{W}_{\tilde{s}}(r)dr\right)\left(\int(r-\frac{1}{2})dW_{\tilde{u}}(r)\right)}{\frac{1}{12}\int \tilde{W}_{\tilde{s}}(r)^2dr - \left(\int(r-\frac{1}{2}\tilde{W}_{\tilde{s}}(r)\right)^2};$$

and $T\{\mu^+(\hat{\beta}^+ - \beta^+) + \mu^-(\hat{\beta}^- - \beta^-)\} = o_P(1)$, *where* $\tilde{W}_{\tilde{s}}(r) := W_{\tilde{s}}(r) - \int W_{\tilde{s}}(r)dr$ *for* $r \in [0, 1]$.

2.2 The Nonlinear ARDL Model

The simple case presented above is useful for exposition and will certainly cover some empirical applications. However, it is too restrictive since it does not allow for weak endogeneity of the regressors and/or serially correlated errors, factors that will significantly affect both the asymptotic and the small sample properties of the estimators. In their presence, the OLS estimator in (2.1) may remain super-consistent but the asymptotic distribution is non-Gaussian. Hence, hypothesis testing cannot be carried out in the usual manner without removing both the serial correlation and the endogeneity of the regressors. In particular, the resulting OLS estimator of the cointegrating parameter will be poorly determined in finite samples.

In the linear cointegration literature, several solutions to these twin problems have been proposed in the context of the static regression model (Phillips and Hansen, 1990; Saikkonen, 1991) and the dynamic regression model (Pesaran and Shin, 1998). Given that our interest is in developing a fully dynamic model, we naturally choose to extend the ARDL approach popularised by Pesaran and Shin (1998) and Pesaran *et al.* (2001), thereby developing a flexible dynamic parametric framework with which to model relationships that exhibit combined long- and short-run asymmetries.[6]

To this end we consider the following nonlinear ARDL(p, q) model:

$$y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \sum_{j=0}^{q} \left(\boldsymbol{\theta}_j^{+\prime} \boldsymbol{x}_{t-j}^{+} + \boldsymbol{\theta}_j^{-\prime} \boldsymbol{x}_{t-j}^{-} \right) + \varepsilon_t, \tag{2.6}$$

where $\boldsymbol{x}_t$ is a $k \times 1$ vector of multiple regressors defined such that $\boldsymbol{x}_t = \boldsymbol{x}_0 + \boldsymbol{x}_t^+ + \boldsymbol{x}_t^-$, ϕ_j is the autoregressive parameter, $\boldsymbol{\theta}_j^+$ and $\boldsymbol{\theta}_j^-$ are the asymmetric distributed-lag parameters, and ε_t is an iid process with zero mean and constant variance, σ_ε^2. Throughout this paper we will focus on the case in which $\boldsymbol{x}_t$ is decomposed into $\boldsymbol{x}_t^+$ and $\boldsymbol{x}_t^-$ around a threshold of zero, thereby distinguishing between positive and negative changes in the rate of growth of $\boldsymbol{x}_t$. The resulting partial sum processes maintain an intuitively appealing and economically meaningful interpretation in a wide range of applications.[7]

Following Pesaran *et al.* (2001), it is straightforward to rewrite (2.6) in the error correction form as

$$\begin{aligned}
\Delta y_t &= \rho y_{t-1} + \boldsymbol{\theta}^{+\prime}\boldsymbol{x}_{t-1}^+ + \boldsymbol{\theta}^{-\prime}\boldsymbol{x}_{t-1}^- + \sum_{j=1}^{p-1}\gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1}\left(\boldsymbol{\varphi}_j^{+\prime}\Delta\boldsymbol{x}_{t-j}^+ + \boldsymbol{\varphi}_j^{-\prime}\Delta\boldsymbol{x}_{t-j}^-\right) + \varepsilon_t \\
&= \rho\xi_{t-1} + \sum_{j=1}^{p-1}\gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1}\left(\boldsymbol{\varphi}_j^{+\prime}\Delta\boldsymbol{x}_{t-j}^+ + \boldsymbol{\varphi}_j^{-\prime}\Delta\boldsymbol{x}_{t-j}^-\right) + \varepsilon_t \qquad (2.7)
\end{aligned}$$

where $\rho = \sum_{j=1}^p \phi_j - 1$, $\gamma_j = -\sum_{i=j+1}^p \phi_i$ for $j = 1, ..., p-1$, $\boldsymbol{\theta}^+ = \sum_{j=0}^q \boldsymbol{\theta}_j^+$, $\boldsymbol{\theta}^- = \sum_{j=0}^q \boldsymbol{\theta}_j^-$, $\boldsymbol{\varphi}_0^+ = \boldsymbol{\theta}_0^+$, $\boldsymbol{\varphi}_j^+ = -\sum_{i=j+1}^q \theta_j^+$ for $j = 1, ..., q-1$, $\boldsymbol{\varphi}_0^- = \boldsymbol{\theta}_0^-$, $\boldsymbol{\varphi}_j^- = -\sum_{i=j+1}^q \theta_j^-$ for $j = 1, ..., q-1$, and $\xi_t = y_t - \boldsymbol{\beta}^{+\prime}\boldsymbol{x}_t^+ - \boldsymbol{\beta}^{-\prime}\boldsymbol{x}_t^-$ is the nonlinear error correction term where $\boldsymbol{\beta}^+ = -\boldsymbol{\theta}^+/\rho$ and $\boldsymbol{\beta}^- = -\boldsymbol{\theta}^-/\rho$ are the associated asymmetric long-run parameters.

To further deal with the possibility of non-zero contemporaneous correlation between the regressors and the residuals in (2.7) we now consider the following reduced form data generating process for $\Delta\boldsymbol{x}_t$:[8]

$$\Delta\boldsymbol{x}_t = \sum_{j=1}^{q-1}\boldsymbol{\Lambda}_j \Delta\boldsymbol{x}_{t-j} + \boldsymbol{v}_t, \qquad (2.8)$$

where $\boldsymbol{v}_t \sim iid\,(0, \boldsymbol{\Sigma}_v)$, with $\boldsymbol{\Sigma}_v$ being a $k \times k$ positive definite covariance matrix. Given our focus on conditional modelling, we may express ε_t conditionally in terms of $\boldsymbol{v}_t$ as:

$$\varepsilon_t = \boldsymbol{\omega}' \boldsymbol{v}_t + e_t = \boldsymbol{\omega}' \left(\Delta \boldsymbol{x}_t - \sum_{j=1}^{q-1} \boldsymbol{\Lambda}_j \Delta \boldsymbol{x}_{t-j} \right) + e_t \tag{2.9}$$

where e_t is uncorrelated with $\boldsymbol{v}_t$ by construction. Substituting (2.9) into (2.7) and rearranging it, we finally obtain the following conditional nonlinear ECM:

$$\Delta y_t = \rho \xi_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta y_{t-j} + \sum_{j=0}^{q-1} \left(\boldsymbol{\pi}_j^{+\prime} \Delta \boldsymbol{x}_{t-j}^{+} + \boldsymbol{\pi}_j^{-\prime} \Delta \boldsymbol{x}_{t-j}^{-} \right) + e_t \tag{2.10}$$

where $\boldsymbol{\pi}_0^{+} = \boldsymbol{\theta}_0^{+} + \boldsymbol{\omega}$, $\boldsymbol{\pi}_0^{-} = \boldsymbol{\theta}_0^{-} + \boldsymbol{\omega}$, $\boldsymbol{\pi}_j^{+} = \boldsymbol{\varphi}_j^{+} - \boldsymbol{\omega}' \boldsymbol{\Lambda}_j$ and $\boldsymbol{\pi}_j^{-} = \boldsymbol{\varphi}_j^{-} - \boldsymbol{\omega}' \boldsymbol{\Lambda}_j$ for $j = 1, ..., q-1$.

It is clear that (2.10) corrects perfectly for the weak endogeneity of any nonstationary explanatory variables and that the choice of an appropriate lag structure will render the model free from residual serial correlation. Our model combines many of the desirable attributes of the fully-modified and the ARDL-based dynamic corrections associated respectively with Phillips and Hansen (1991) and Pesaran and Shin (1998) in a dynamic parametric framework capable of modelling both long- and short-run asymmetries. Moreover, since our model is linear in all the parameters including $\boldsymbol{\theta}^{+}$, $\boldsymbol{\theta}^{-}$, $\boldsymbol{\pi}_i^{+}$ and $\boldsymbol{\pi}_i^{-}$, reliable estimation of (2.10) can be achieved by standard OLS.

Following the conditions used in the derivations above, we now summarise the following assumption in the context of the NARDL-based ECM, (2.10):

Assumption 2 *(i) $e_t \sim iid(0, \sigma_e^2)$; (ii) $\boldsymbol{x}_t$ is a $k \times 1$ vector of I(1) regressors given by (2.8); (iii) e_t is uncorrelated with $\boldsymbol{v}_t$ through the conditional modelling, (2.9); (iv) the condition, $\rho < 0$ guarantees that the model is dynamically stable.*

Following Theorems 3.1 and 3.2 in Pesaran and Shin (1998), it is straightforward to show under Assumption 2 that: (i) the OLS estimators of all the short-run dynamic parameters in (2.10) are $\sqrt{T}$-consistent and have the asymptotic normal distribution, and (ii) the OLS estimators of the long-run parameters computed as $\hat{\boldsymbol{\beta}}^{+} = -\hat{\boldsymbol{\theta}}^{+}/\hat{\rho}$ and $\hat{\boldsymbol{\beta}}^{-} = -\hat{\boldsymbol{\theta}}^{-}/\hat{\rho}$, are T-consistent and follow the mixture normal distribution as defined in Theorem 1. Hence, the null hypotheses of a symmetric long-run relationship $\left(\boldsymbol{\beta}^{+} = \boldsymbol{\beta}^{-}\right)$ or symmetric short-run coefficients can be tested using the Wald statistic following an asymptotic χ^2 distribution. In order to assess the extent to which these theoretical predictions are validated in both large and small samples, we will conduct a series of Monte Carlo experiments in Section 3.

2.3 Bounds-Testing the Asymmetric Long-Run Relationship

We develop two operational testing procedures for the existence of an asymmetric (cointegrating) long-run relationship based on the NARDL ECM, (2.10). If $\rho = 0$, (2.10) reduces to the regression involving only first differences, implying that there is no long-run relationship between the levels of y_t, $\boldsymbol{x}_t^{+}$ and $\boldsymbol{x}_t^{-}$. We first follow Banerjee *et al.* (1998) and propose the t-statistic testing $\rho = 0$ against $\rho < 0$ in (2.10). Next, we follow Pesaran, Shin and Smith (2001) and propose an F-test of the joint null, $\rho = \boldsymbol{\theta}^{+} = \boldsymbol{\theta}^{-} = 0$ in (2.10). We denote these tests, t_{BDM} and F_{PSS}, respectively.

The asymptotic distributions of these test statistics are non-standard under their respective null hypotheses and their exact asymptotic distributions are generally complicated to derive due to the complex dependence structure between $\boldsymbol{x}_t^{+}$ and $\boldsymbol{x}_t^{-}$, especially when the means of Δy_t and $\Delta \boldsymbol{x}_t$ are non-zero.[9] In light of these difficulties, we propose the use of the pragmatic 'bounds-testing' approach advanced by Pesaran *et al.* (2001). Two extreme cases can be identified, one in which the level regressors $\boldsymbol{x}_t^{+}$ and $\boldsymbol{x}_t^{-}$ in (2.10) are all $I(1)$, and the other in which they are all $I(0)$. It follows that critical values tabulated for these two scenarios provide critical value bounds for all classifications, irrespective of whether the regressors are $I(0)$, $I(1)$ or mutually cointegrated. This approach is particularly useful in the current context due to the various dependence structures (including cointegration) that may exist between $\boldsymbol{x}_t^+$ and $\boldsymbol{x}_t^-$. Following Pesaran *et al.* (2001), we differentiate between five cases of (2.10) for the F_{PSS} statistic: (i) without intercept or linear trend; (ii) with restricted intercept only; (iii) with unrestricted intercept only; (iv) with intercept and restricted linear trend; and (v) with intercept and unrestricted linear trend. Similarly, for the t_{BDM} statistic we differentiate between cases (i), (iii) and (v). Pesaran *et al.* (2001) tabulate the critical value bounds for both the F_{PSS} and t_{BDM} statistics under each of these cases for a range of values of k, the number of regressors entering the long-run relationship.

In the context of the NARDL model, due to the dependence structure that exists between the partial sum decompositions $\boldsymbol{x}_t^+$ and $\boldsymbol{x}_t^-$, the exact value of k is not clear. In the simplest case where the long-run relationship is defined between y_t, x_t^+ and x_t^-, it follows that the true value of k lies between 1 and 2.[10] In general, we expect that the test will be modestly undersized using $k = 1$ and similarly oversized with $k = 2$. Employing the $k = 1$ critical values results in a more conservative test (a higher critical value) so, at a pragmatic level, rejecting the null of no long-run relationship using these critical values provides strong evidence of the existence of a long-run relationship. The mis-sizing of the test can be readily resolved by bootstrapping, although in practice we find that the pragmatic approach typically leads to the same conclusion. This observation is reinforced below by a series of Monte Carlo simulation experiments designed to evaluate the finite sample properties of the PSS test and the associated bootstrapping routine.

2.4 Asymmetric Dynamic Multipliers

It is straightforward to derive asymmetric dynamic multipliers associated with unit changes in $\boldsymbol{x}_t^+$ and $\boldsymbol{x}_t^-$, respectively, on y_t. Consider the ARDL-in-levels representation of (2.10):

$$\phi(L)\, y_t = \boldsymbol{\theta}^+(L)\, \boldsymbol{x}_t^+ + \boldsymbol{\theta}^-(L)\, \boldsymbol{x}_t^- + e_t, \tag{2.11}$$

where $\phi(L) = 1 - \sum_{i=1}^{p-1} \phi_i L^i$, $\boldsymbol{\theta}^+(L) = \sum_{i=0}^{q} \boldsymbol{\theta}_i^+ L^i$, and $\boldsymbol{\theta}^-(L) = \sum_{i=0}^{q} \boldsymbol{\theta}_i^{-1} L^i$.[11] Premultiplying (2.11) by the inverse of $\phi(L)$, we obtain:

$$y_t = \boldsymbol{\lambda}^+(L)\, \boldsymbol{x}_t^+ + \boldsymbol{\lambda}^-(L)\, \boldsymbol{x}_{t-i}^- + [\phi(L)]^{-1} e_t, \tag{2.12}$$

where $\boldsymbol{\lambda}^+(L) \left(= \sum_{j=0}^{\infty} \boldsymbol{\lambda}_j^+\right) = \phi(L)^{-1}\, \boldsymbol{\theta}^+(L)$ and $\boldsymbol{\lambda}^-(L) \left(= \sum_{j=0}^{\infty} \boldsymbol{\lambda}_j^-\right) = \phi(L)^{-1}\, \boldsymbol{\theta}^-(L)$.[12] The cumulative dynamic multiplier effects of $\boldsymbol{x}_t^+$ and $\boldsymbol{x}_t^-$ on y_t can be evaluated as follows:

$$\boldsymbol{m}_h^+ = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial \boldsymbol{x}_t^+} = \sum_{j=0}^{h} \boldsymbol{\lambda}_j^+,\ \ \boldsymbol{m}_h^- = \sum_{j=0}^{h} \frac{\partial y_{t+j}}{\partial \boldsymbol{x}_t^-} = \sum_{j=0}^{h} \boldsymbol{\lambda}_j^-,\ \ h = 0, 1, 2... \tag{2.13}$$

Notice that, by construction, as $h \to \infty$, $\boldsymbol{m}_h^+ \to \boldsymbol{\beta}^+$ and $\boldsymbol{m}_h^- \to \boldsymbol{\beta}^-$, where $\boldsymbol{\beta}^+ = -\boldsymbol{\theta}^+/\rho$ and $\boldsymbol{\beta}^- = -\boldsymbol{\theta}^-/\rho$ are the asymmetric long-run coefficients. There is little reason to believe that the dynamic adjustment patterns summarised by $\boldsymbol{m}_h^+$ and $\boldsymbol{m}_h^-$ should generally be symmetric. Therefore, even though we do not directly model asymmetric error correction (*i.e.* we do not allow for regime-dependency of ρ in (2.10)) we may still observe asymmetric adjustment paths and/or duration of the disequilibrium. This highlights an important feature of the NARDL model. In the interest of clarity, when discussing asymmetry we tend to distinguish only between long- and short-run asymmetries. However, the NARDL model in fact admits *three* general forms of asymmetry: (i) long-run or reaction asymmetry, associated with $\boldsymbol{\beta}^+ \neq \boldsymbol{\beta}^-$; (ii) impact asymmetry, associated with the inequality of the coefficients on the contemporaneous first differences Δx_t^+ and Δx_t^-; (iii) adjustment asymmetry, captured by the patterns of adjustment from initial equilibrium to the new equilibrium following an economic perturbation (*i.e.* the dynamic multipliers). Adjustment asymmetry derives from the interaction of impact and reaction asymmetries in conjunction with the error correction coefficient, ρ.

In practice, the patterns of dynamic adjustment will depend on the model specification. Four distinct cases can be identified: the unrestricted specification, (2.10), accommodating asymmetries in both the short- and long-run and three restricted specifications obtained by imposing short- and long-run symmetry restrictions in (2.10), either separately or jointly. An early study by Borenstein *et al.* (1997) investigates short-run dynamic asymmetries in the response of retail gasoline prices to fluctuations in the price of crude oil by implicitly imposing the long-run symmetry restrictions $\boldsymbol{\theta}^+ = \boldsymbol{\theta}^- = \boldsymbol{\theta}$ such that (2.10) simplifies to[13]

$$\Delta y_t = \rho y_{t-1} + \boldsymbol{\theta x}_{t-1} + \sum_{i=1}^{p-1} \gamma_i \Delta y_{t-i} + \sum_{i=0}^{q-1} \left(\boldsymbol{\pi}_i^+ \Delta \boldsymbol{x}_{t-i}^+ + \boldsymbol{\pi}_i^- \Delta \boldsymbol{x}_{t-i}^- \right) + e_t. \tag{2.14}$$

Models of this form have also been employed by Shirvani and Wilbratte (2000) and Apergis and Miller (2006) in their analysis of short-run asymmetric wealth effects on consumption due to liquidity constraints.

Short-run symmetry restrictions can take either of two forms: (i.) $\boldsymbol{\pi}_i^+ = \boldsymbol{\pi}_i^-$ for all $i = 0, ..., q-1$ or (ii.) $\sum_{i=0}^{q-1} \boldsymbol{\pi}_i^+ = \sum_{i=0}^{q-1} \boldsymbol{\pi}_i^-$. When imposing such restrictions in the presence of an asymmetric long-run relationship, we obtain:[14]

$$\Delta y_t = \rho y_{t-1} + \boldsymbol{\theta}^+ \boldsymbol{x}_{t-1}^+ + \boldsymbol{\theta}^- \boldsymbol{x}_{t-1}^- + \sum_{i=1}^{p-1} \gamma_i \Delta y_{t-i} + \sum_{i=0}^{q-1} \boldsymbol{\pi}_i \Delta \boldsymbol{x}_{t-i} + e_t. \tag{2.15}$$

Finally, the most restrictive specification is obtained when assuming linearity of the long-run relationship in conjunction with symmetric short-run adjustment:

$$\Delta y_t = \rho y_{t-1} + \boldsymbol{\theta x}_{t-1} + \sum_{i=1}^{p-1} \gamma_i \Delta y_{t-i} + \sum_{i=0}^{q-1} \boldsymbol{\pi}_i \Delta \boldsymbol{x}_{t-i} + e_t. \tag{2.16}$$

It is clear that (2.14), (2.15) and (2.16) are special cases of the unrestricted specification described by (2.10) and that the long- and short-run symmetry restrictions can be easily tested in the usual manner following our proposed methodology. Our early experimentation with the model, as well as the results adduced in Van Treeck (2008), Nguyen and Shin (2010) and Greenwood-Nimmo, Shin and Van Treeck (2011), suggest that the dynamic multipliers obtained from the various cases are generally significantly different from one-another. Moreover, it is generally the case that the results of linear estimation are profoundly misleading when the underlying relationship is, in fact, asymmetric. This will become apparent during the discussion of our empirical illustrations in Section 4.

A simple and useful addition to the general typology developed above is the extension to the case where a subset of regressors enters the long-run relationship symmetrically:[15]

$$y_t = \boldsymbol{\beta}^{+\prime} \boldsymbol{x}_t^+ + \boldsymbol{\beta}^{-\prime} \boldsymbol{x}_t^- + \boldsymbol{\gamma}' \boldsymbol{w}_t + u_t, \tag{2.17}$$

where $\boldsymbol{x}_t \left(= \boldsymbol{x}_0 + \boldsymbol{x}_t^+ + \boldsymbol{x}_t^-\right)$ is a $k \times 1$ vector of regressors entering the model asymmetrically and $\boldsymbol{w}_t$ is a $g \times 1$ vector of regressors entering symmetrically. Extending the concept of partial asymmetry to both the long- and short-run within our NARDL model, we obtain:

$$\begin{aligned} \Delta y_t \;\; = \;\; & \rho y_{t-1} + \boldsymbol{\theta}^+ \boldsymbol{x}_{t-1}^+ + \boldsymbol{\theta}^- \boldsymbol{x}_{t-1}^- + \boldsymbol{\theta}_w \boldsymbol{w}_{t-1} \\ & + \sum_{i=1}^{p-1} \gamma_i \Delta y_{t-i} + \sum_{i=0}^{q-1} \left(\boldsymbol{\pi}_i^+ \Delta \boldsymbol{x}_{t-i}^+ + \boldsymbol{\pi}_i^- \Delta \boldsymbol{x}_{t-i}^- + \boldsymbol{\pi}_{w,i} \Delta \boldsymbol{w}_{t-i} \right) + e_t. \end{aligned} \tag{2.18}$$

In light of the bounds-testing approach employed above, it follows that estimation and inference proceed exactly as before, irrespective of whether $\boldsymbol{x}_t$ and $\boldsymbol{w}_t$ are $I(0)$, $I(1)$ or mutually cointegrated. Furthermore, it is once again clear that this partially asymmetric form represents a special case of (2.10).

3 Finite Sample Properties

In order to investigate the finite sample properties of the estimators we conduct a range of Monte Carlo experiments based on the following simple data generating process (DGP):

$$\Delta y_t = a + \rho \left(y_{t-1} - \beta^+ x_{t-1}^+ - \beta^- x_{t-1}^- \right) + \varphi^+ \Delta x_t^+ + \varphi^- \Delta x_t^- + u_t, \tag{3.19}$$

where $\Delta x_t = \varepsilon_t$, and (u_t, ε_t) are serially uncorrelated and are generated according to the following bivariate normal distribution:

$$\begin{pmatrix} u_t \\ \varepsilon_t \end{pmatrix} \sim N \left\{ \mathbf{0}, \mathbf{\Omega} = \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix} \right\}. \tag{3.20}$$

Notice that when $\omega \neq 0$, (3.19) can be estimated by:

$$\Delta y_t = a + \rho y_{t-1} + \theta^+ x_{t-1}^+ + \theta^- x_{t-1}^- + \pi^+ \Delta x_t + \pi^- \Delta x_t^- + e_t, \tag{3.21}$$

where $\pi^+ = \varphi^+ + \omega$ and $\pi^- = \varphi^- + \omega$ and the long run parameters are defined as $\hat{\beta}^+ = -\hat{\theta}^+/\hat{\rho}$ and $\hat{\beta}^- = -\hat{\theta}^-/\hat{\rho}$.

We experiment with a wide variety of parameterisations of (3.19) and (3.20). Specifically, under the assumptions that $a = 0$, $\beta^+ = 0.5$ and $\varphi^+ = 0.5$, and denoting $\beta^- = \beta^+ + \delta_\beta$ and $\varphi^- = \varphi^+ + \delta_\varphi$, we experiment with an array of combinations of the following parameters: $\rho \in (-0.05, -0.1, -0.2)$, $\delta_\beta \in (0.1, 0.2, 0.25, 0.5)$, $\delta_\varphi \in (0.1, 0.2, 0.25, 0.5)$, $\omega \in (-0.5, 0, 0.5)$, and $T \in (100, 200, 400)$. Due to space constraints, we are unable to report the results of all of these simulations herein[16]. Rather, we summarise the key findings that arise across these parameterisations and report in detail the results from a baseline case in which we use $\rho = -0.2$, $\delta_\beta = 0.5$ and $\delta_\varphi = 0.5$, and where ω and T vary over the ranges defined above.

In Table 1 we report a range of summary statistics for the parameter estimates based on our simulations using 3,000 replications of our baseline case. We note that the bias and error in the estimation of each of the parameters is largely negligible (this also holds under the other parameterisations of the DGP that we consider). The only exception to this generalisation is the error correction parameter, which shows a modest downward bias especially when $T \leq 100$. However, this observation is not unexpected given the well-documented downward bias associated with the estimation of AR(1) coefficients in time series models.

– TABLE 1 ABOUT HERE –

We also investigate the finite sample size and power of the Wald statistics for the null hypothesis of long-run symmetry $\left(H^S_{LR} : \beta^+ = \beta^-\right)$ and the null of symmetric short-run dynamics $\left(H^S_{SR} : \pi^+ = \pi^-\right)$. To this end, we consider the model for H^S_{LR}:

$$\Delta y_t = a + \rho \left(y_{t-1} - \beta x_{t-1}\right) + \varphi^+ \Delta x^+_t + \varphi^- \Delta x^-_t + u_t, \tag{3.22}$$

where we set $\beta = \beta^+$, and the model for H^S_{SR}:

$$\Delta y_t = a + \rho \left(y_{t-1} - \beta^+ x^+_{t-1} - \beta^- x^-_{t-1}\right) + \varphi \Delta x_t + u_t, \tag{3.23}$$

where $\varphi = \varphi^+$. In both cases the alternative model is given by (3.19). Finally, we examine the finite sample size and power of the PSS bounds test of the null hypothesis of no asymmetric cointegration ($H_{PSS} : \rho = \beta^+ = \beta^- = 0$). In this case, the restricted model is given by:

$$\Delta y_t = a + \varphi^+ \Delta x_t^+ + \varphi^- \Delta x_t^- + u_t. \tag{3.24}$$

and, as before, the alternative model is given by (3.19). As noted in Section 2.3, the relevant critical value bounds for the PSS test depend on the number of regressors entering the long-run relationship, k. However, given the dependence between $\boldsymbol{x}_t^+$ and $\boldsymbol{x}_t^-$, the appropriate value of k is unclear. Thus, we propose a pragmatic solution using two sets of critical values, one for which k is defined by counting the partial sums as separate $I(1)$ regressors (here, $k = 2$) and another by counting each set of partial sums collectively as a single $I(1)$ regressor (here, $k = 1$). It follows that the latter approach is the more conservative.

Table 2 summarises the simulation results from our baseline case at a nominal size of 5%. For $T = 100$, the long-run Wald test has very high power and the short-run Wald and PSS tests have moderate power, although this rapidly improves as T increases. Indeed, when $T = 400$ all of the tests achieve close to 100% power. The short-run Wald test is well-sized regardless of the value of T while W_{LR} is slightly oversized in small samples, although this improves rapidly as T increases. Finally, as expected, we observe some mis-sizing of the PSS F-test dependent on the selection of k. Importantly, however, we find that the power of the test is satisfactory even under the conservative case ($k = 1$).

– TABLE 2 ABOUT HERE –

Table 2 also reports the power of the bootstrapped PSS test. For each replication of the simulation routine, using data generated under the alternative hypothesis, we generate 500 bootstrap samples non-parametrically using the resampled residuals from estimation of (3.21) in conjunction with the estimated coefficients from (3.24) under the assumption that the initial values and the $\boldsymbol{x}$'s are known. It is then a simple matter to compute the empirical p-value of the PSS test by estimating (3.21) on the bootstrap samples and calculating the probability that the bootstrapped test statistic exceeds its original value. On this basis, we note that the bootstrapping procedure achieves the desired size correction while retaining admirable power which increases with T.

One important finding that arises from the other parameterisations of the DGP is that the power of the long- and short-run Wald tests is positively associated with the distance between their respective null and alternative hypotheses. Moreover, we find that the long-run Wald test becomes somewhat over-sized especially when the distance of the alternative from the null is small, the error correction parameter is close to zero, and $T \leq 100$. These findings reflect the well known limitations of asymptotic inference under adverse conditions. To overcome these issues, one could adopt the common practice within the literature and compute empirical p-values for the short- and long-run Wald statistics by use of a bootstrap. However, we choose to pursue an alternative and more flexible approach. By computing 95% bootstrap confidence intervals for the difference between the asymmetric cumulative dynamic multipliers defined for positive and negative shocks, respectively, we are able to convey relevant information about the statistical significance of any observed asymmetries at any horizon, h, and over any timeframe $h_1 \leq h \leq h_2$. Furthermore, in light of our simulations, and given the absence of precise asymptotic critical values for the F_{PSS} and t_{BDM} test statistics, we choose to provide bootstrapped p-values for these tests in our empirical applications.[17]

4 Empirical Applications

To demonstrate both the simplicity and the flexibility of the NARDL approach, we will present two empirical applications. Firstly, we will examine nonlinearities in the bivariate relationship between output and unemployment in the US, Canada and Japan. Secondly, we will apply our technique to the trivariate case of gasoline pricing in Korea.

4.1 Asymmetric Unemployment-Output Relationship

The negative relationship between changes in the rate of unemployment and the rate of output growth (Okun's Law) remains one of the most commonly cited stylized facts in modern macroeconomics. It is of fundamental importance in monetary policy transmission, representing the link between unemployment and output which underpins the mechanism by which inflation targeting monetary policy is thought to operate.

However, despite its importance, empirical assessments of Okun's law over the last three decades have been rather disappointing. The majority of this voluminous literature adheres to a linear paradigm, reflecting the assumption that cyclical upturns and downturns have symmetrical effects on unemployment. In general, there is little reason to believe that the labour market should behave in this simplistic fashion. If employers dismiss a given quantity of labour after a negative growth shock, then they may not hire exactly the same amount after a positive shock of equal magnitude (Lang and de Peretti, 2009). This may be discussed in terms of labour market hysteresis, the idea that cyclical shocks may permanently affect structural unemployment. In this vein, Blanchard and Summers (1987) explain the persistently high European unemployment of the 1980s using an insider-outsider wage setting model. They argue that adverse shocks that reduce the proportion of insiders (union members) will increase outsider unemployment permanently. There is, therefore, no tendency for the labour market to return to its initial state even after economic growth has recovered (see also Hammermesh and Pfann, 1998, on the asymmetric adjustment costs of labour).

In response to these issues, empirical attention is increasingly turning to nonlinear modelling. There is a natural complementarity between the asymmetric analyses of Okun's Law, the Phillips curve and the preferences of the central bank which has helped to drive research in the field. Neftci (1984) laid the foundations for this literature with his early study of business cycle effects on the patterns of correlation between major US time series, which revealed that the output-unemployment relationship displays marked asymmetry. Altissimo and Violante (2001) find evidence of nonlinearity between output and unemployment using a nonlinear multivariate VAR model. Their results, which they note are consistent with the majority of existing univariate threshold models, indicate that shocks in the recessionary regime are considerably less persistent than those in the expansionary regime. Similarly, Crespo Cuaresma (2003) develops a regime-dependent specification of Okun's law and finds that the contemporaneous effect of output growth on unemployment is asymmetric and significantly larger in recessions than in expansions, and that shocks to unemployment tend to be more persistent in the expansionary regime.

Attfield and Silverstone (1998) argue that if output and unemployment are cointegrated and potential output and unemployment are defined by the stochastic trend components of the variables constructed from the Beveridge-Nelson decomposition, then Okun's coefficient can be interpreted as the cointegrating coefficient. However, the cointegration test results are ambiguous: the single equation residual based ADF test is unable to reject the null of no cointegration while it is rejected by the Johansen test. Using a static asymmetric regression of the form of (2.1), Schorderet (2001) finds that nonlinearity hinders efforts to detect the stationary relationship between unemployment and output.[18] The contention that the appropriate modelling of nonlinearity strongly affects the cointegration test is one to which we will return shortly.

In this section, we apply the NARDL technique to the simultaneous analysis of both long- and short-run nonlinearities in the relationship between output and unemployment in the US, Canada and Japan.[19] This application demonstrates one of the key strengths of our model: its flexibility and the ease with which it can be applied to each of the four cases of nonlinearity defined above.

Firstly, to establish a reference point, we estimate the static linear regression of unemployment on a constant, a time trend and output (Table 3(a)) and a static asymmetric model of the form of (2.1), the results of which are reported in Table 3(b).

– TABLE 3 ABOUT HERE –

In keeping with the findings of Attfield and Silverstone (1998), Schorderet (2001) and Granger and Yoon (2002), the EG test finds no evidence of linear cointegration. Moreover, the EG test is unable to reject the null of no cointegration in the static asymmetric case, highlighting the importance of an appropriate dynamic specification. In all cases, we find a pronounced negative association between output and unemployment, with the results of asymmetric analysis indicating strong non-linearity (the Wald tests reject the null in all cases). However, the validity of these results is questionable given the evidence of severe model mis-specifications.

Table 4 reports estimation results for the restricted symmetric ARDL regression of the form of (2.16). Table 5 presents the results of the unrestricted NARDL case allowing for both long- and short-run asymmetry. Notice that the cointegration tests are unable to reject the null hypothesis in the restricted case but that both the t_{BDM} and F_{PSS} statistics resoundingly reject the null when long-run asymmetry is modelled appropriately. This result underscores the importance of correctly specifying the long-run relationship under scrutiny. Moreover, the finding that the ECM-based tests are able to detect the asymmetric long-run relationship while the EG residual-based approach cannot is generally consistent with the works of Kremers, Ericsson and Dolado (1992), Hansen (1995), Banerjee *et al.* (1998) and Pesaran *et al.* (2001). This reflects the well-established power-dominance of the ECM-based tests resulting from their inclusion of potentially valuable information relating to the correlation between the regressors and the underlying disturbances.

– TABLES 4 & 5 ABOUT HERE –

In the restricted symmetric models (Table 4), the estimated long-run coefficients for the US, Canada and Japan are -1.66, -5.68 and 5.57, respectively, although none is statistically significant due to the failure to accurately model the long-run relationship. Indeed, the counterintuitive finding of a positive long-run coefficient in the case of Japan reflects the fact that the model misspecification is so severe in this case that the estimated error correction coefficient is positive, indicating explosive instability. By contrast, using the more general unrestricted model of the form (2.10), the F_{PSS} and t_{BDM} tests both reject their respective null hypotheses in all cases, even using the conservative critical values for the PSS test (see Table 5). Furthermore, the Wald tests are also able to firmly reject the null hypothesis of long-run symmetry in all cases. In this case, the estimated long-run coefficients on y^+ and y^- are -9.76 and -28.88 for the US, -17.26 and -28.48 for Canada and -7.28 and -11.26 for Japan, respectively. Therefore, we may conclude that an economic upturn of 10.3% is necessary to reduce unemployment by 1% in the US while an economic downturn of just 3.5% achieves the opposite. The associated values for Canada are 5.8% and 3.5% while in the case of Japan the figures translate to an economic upturn of 13.7% and a downturn of 8.9%. The relatively muted response of the labour market to output fluctuations in Japan reflects its restrictive employment policies and unusually long job tenure (Tanaka, 2001), and is comparable to the linear estimation results achieved by Hamanda and Kurosaka (1984).

Turning to the analysis of short-run dynamic asymmetry, we find that the Wald test cannot reject the null of (weak-form) summative symmetric adjustment in the USA or Japan but that it is rejected at the 10% level in Canada. Consulting the bootstrap confidence intervals for the difference between the asymmetric dynamic multipliers reported in Figures 1–3 supports this finding. However, as noted earlier, the pattern of dynamic adjustment depends on a combination of the long-run parameters, the error correction coefficient and the model dynamics. Therefore, although we find little evidence of additive short-run asymmetries, we nevertheless observe apparent asymmetries in the adjustment patterns traced by the dynamic multipliers.

– FIGURES 1 – 3 ABOUT HERE –

For the benefit of the reader, Figure 1 presents the dynamic multipliers for the US under each of the four combinations of long- and short-run asymmetry. Notice that the imposition of long-run symmetry restrictions fundamentally changes the shape of the dynamic multipliers, resulting in marked overshooting where none was previously observed. In conjunction with the results of a battery of diagnostic tests, we conclude that the imposition of invalid long-run restrictions represents a severe mis-specification of the model. This underscores the importance of correctly accounting for inherent nonlinearities in the long-run relationship and cautions that failure to do so jeopardises the identification of the long-run relationship and compromises the estimation of the model dynamics. In light of the overwhelming rejection of the long-run symmetric models, the associated dynamic multipliers are omitted from Figures 2 and 3 to save space.

For the US, the results of both long-run asymmetric models (Figures 1(a) and (c)) are remarkably similar, indicating that the labour market responds rapidly and strongly to cyclical downturns in the very short-run (correcting one quarter of disequilibrium within one period) but that full adjustment to the new equilibrium is a relatively prolonged process. By contrast, the labour market responds only mildly to the boom phase but full adjustment is achieved within six months. This reflects the flexibility of the US labour market, whereby firms are quick to fire in the short-run in order to cut costs but are also quick to hire in the knowledge that they can easily and quickly release the additional labour should the need arise.

Figure 2 reveals that the pattern of dynamic adjustment is considerably richer in the fully asymmetric case in Canada. We again find very rapid labour market adjustment in the immediate wake of a recessionary shock, with more than 50% of the traverse to equilibrium achieved within six months. Again, we find that the remaining disequilibrium error is corrected relatively slowly. By contrast, the labour market response to the cyclical upswing is more gradual, taking one year to achieve 50% of the adjustment toward equilibrium. Furthermore, in panel (b), with the imposition of short-run symmetry, after the initial rapid adjustment to the recessionary shock the gradient of the cumulative dynamic multiplier is noticeably steeper than in the case of an economic expansion, as reflected in the upward slope of the difference curve. In sum, our results suggest that Canadian firms are quick to fire and slow to hire, reflecting conservatism on the part of their management.

Finally, we find little evidence of short-run asymmetry in Japan. Figure 3 reveals that the Japanese labour market exhibits very muted responses to both booms and busts when compared to the US and Canada, a finding that reflects the prevalence of restrictive labour market institutions. Focusing on Figure 3(b), we note that 50% of the equilibrium correction occurs within 10-12 months of either a positive or a negative shock, and that after this initial phase, convergence upon long-run equilibrium occurs very slowly.

Despite their superficial differences, a common pattern emerges between Figures 1, 2 and 3. In general, the labour markets in all countries exhibit relatively rapid adjustment in the first year with the absolute effect of an economic contraction being significantly larger than that of an expansion. Following this initial period, the speed of adjustment slows markedly, and subject to the imposition of short-run symmetry restrictions, we find that the labour market response to output shocks remains somewhat more rapid in the recessionary case than in the expansionary environment in both Canada and Japan. The US can be viewed as a special case due to the widely discussed flexibility of its labour market which permits very rapid adjustment to the expansionary shock as firms are eager to hire in the knowledge that subsequent dismissals are neither difficult nor unduly costly.

The subtle patterns revealed by the dynamic multipliers suggest that the focus of the literature on the persistence of shocks (Altissimo and Violante, 2001; Crespo Cuaresma, 2003) fails to convey important information regarding the magnitude of the implied adjustments to the labour market. Simply put, the impact of a recession in terms of jobs lost is greater in both the short- and the long-run than the job creation associated with an economic expansion of equal magnitude even though the discussion of the half-life of the shocks in the US may indicate the opposite (*i.e.* 50% of the long-run effect of a recessionary shock is greater than 100% of the long-run impact of an expansionary shock of equal magnitude). Focusing on persistence gives an incomplete picture of the phenomenon under study when the long-run relationship is asymmetric. This serves to highlight one of the primary attributes of the asymmetric cumulative dynamic multipliers; they help to shed light on the traverse between the short-run and the long-run, a property whose usefulness and theoretical appeal is difficult to overstate. In a traditional ECM, the speed of adjustment is computed simply as a percentage of the equilibrium error that is corrected in each period. By contrast, NARDL illuminates the dynamic pattern of adjustment in a simple and intuitive manner.

4.2 Asymmetric Gasoline Price Adjustment in Korea

A large literature has developed around the observation that retail gasoline prices tend to react asymmetrically to changes in the price of crude oil (an exhaustive survey is provided by Grasso and Manera, 2007). This phenomenon has come to be referred to as the 'rockets and feathers' hypothesis following the early contribution of Bacon (1991). Employing an asymmetric partial adjustment model in which the adjustment process is assumed to be quadratic, Bacon's results support the hypothesised asymmetry. Similarly, Borenstein *et al.* (1997, BCG) derive strong support for asymmetry from a hybrid error correction model where changes in gasoline and oil prices are decomposed into positive and negative changes.[20]

Various theoretical explanations for asymmetric price adjustment have been adduced in the literature, the dominant three being oligopolistic pricing behaviour (Radchenko, 2005), inventory capacity and costs (Borenstein and Shepard, 2002) and nonlinear consumer search-effort (Johnson, 2002). While the literature on short-run dynamic asymmetry is expansive, relatively little work has been done on potential long-run asymmetry.[21]

Reilly and Witt (1998) were among the first authors to investigate asymmetric pass-through of the exchange rate to the retail price of gasoline, reflecting the convention of quoting oil prices in US$ per barrel. Their results, derived from a simple ECM specification encompassing short-run asymmetries, support the hypothesis of a non-linear relationship between the exchange rate and retail gasoline prices for the UK. The authors report that a Sterling depreciation is rapidly passed through to higher prices at the pump but that a strengthening of the Pound is not met by a commensurate reduction in retail prices. Similarly, Asplund, Eriksson and Friberg (2000) find that the impact of a depreciation is more marked than that of an appreciation in Sweden, with retail gasoline prices reacting more swiftly to the exchange rate than to crude oil price movements. More recently, Galeotti, Lanza and Manera (2003) find compelling evidence that the speed of adjustment to long-run equilibrium is asymmetric both with respect to oil price shocks and exchange rate shocks. However, these papers consider only short-run dynamic asymmetries and abstract from long-run nonlinearity.

The majority of papers surveyed by Grasso and Manera (2007) rely on the two-step Engle-Granger estimation technique in which linear homogeneity of the long-run relationship is imposed in the first step. This methodology is only appropriate in the analysis of short-run asymmetry where the long-run relationship is believed to be linear.[22] Should the underlying long-run relationship prove nonlinear, the imposition of linearity in the first step is likely to provide misleading and spurious results as noted in the case of Okun's Law above. We contribute to this literature by applying our modelling strategy to the case of asymmetric pass-through of crude oil price changes and exchange rate fluctuations to the retail price of gasoline in Korea over the period 1991q1-2007q2.[23] The choice of Korean data is motivated by the need to find an industrial country which is entirely reliant on imported oil, thereby circumventing any issues of endogeneity of regressors that may arise in countries with significant oil extraction and refining activity. Given the extensive literature surveyed above, we do not report static estimation results and merely note that such simple models tend to exhibit profound evidence of misspecification.

Table 6 presents the results of the benchmark symmetric ARDL model, the fully asymmetric NARDL model and our preferred specification which combines long-run symmetry with short-run asymmetry. Taken together, the F_{PSS} and t_{BDM} tests indicate cointegration in both of the asymmetric models.[24]

– TABLE 6 & FIGURE 4 ABOUT HERE –

The Wald tests fail to reject long-run symmetry with respect to either the oil price or the exchange rate, indicating that the pass-through from input prices to the retail price of gasoline is linear in the long-run. This may suggest that the retail gasoline industry in Korea has been relatively competitive. Alternatively, it may be attributed to state intervention in the energy industry in the early years of the sample. Turning to the short-run, the Wald tests decisively reject the null of additive short-run symmetry with respect to both the oil price and the exchange rate. This pattern of asymmetry determines the shape of the dynamic multipliers presented in Figure 4. Focusing first on the retail price response to the crude oil spot price, we observe a strong and rapid reaction to positive changes but a more gradual response to falling crude prices in both panels (a) and (b). The principle difference between these two figures derives from the considerable uncertainty surrounding the long-run coefficient estimates in the fully asymmetric model.

This inflates the bootstrap confidence intervals in panel (a) but, interestingly, also seems to exaggerate the observed short-run asymmetry. In conjunction with the weight of evidence supporting the rockets and feathers hypothesis, we therefore regard the combination of long-run symmetry and short-run asymmetry as the most plausible case.

Turning to the case of exchange rate fluctuations, we again note that the long-run symmetry restrictions cannot be rejected but that the additive short-run restrictions are firmly rejected. Figures 4(c) and (d) reveal that gasoline prices increase rapidly and strongly following a weakening of the Korean Won, displaying mild overshooting. By contrast, the response to an appreciation is rather muted. Moreover, our results suggest that exchange rate fluctuations have a more pronounced impact on retail gasoline prices than movements in the price of crude oil quoted in US$. This effect is apparent in both the long- and the short-run and is consistent with the findings of Asplund *et al.* (2000).

Overall, our results are largely consistent with the existing literature on dynamic asymmetries, confirming that Korean gasoline prices respond more rapidly to the price increases of crude oil than to decreases and that they are more sensitive to exchange rate depreciations than to appreciations. By contrast, little research concerning long-run asymmetries exists against which to judge our results. However, at a pragmatic level, one can argue that the presence of long-run asymmetries in the gasoline-pricing equation may give rise to a logical inconsistency, and so our finding of long-run symmetry may be considered theory-consistent. If there was a persistent long-run tendency for gasoline prices to increase more following a depreciation than they would decrease following an appreciation of equal magnitude, for example, then there would be a ratchet mechanism at work whereby prices would gradually increase through time under the assumption that positive and negative shocks are of approximately equal magnitude and probability. This outcome seems rather implausible and suggests that long-run linearity is the more natural case.

5 Concluding Remarks

The investigation of nonstationarity in conjunction with nonlinearity has recently assumed a prominent role in econometric research. This reflects the realisation that asymmetry is pervasive within the social sciences and may be inherent in modern economies. Indeed, the behavioural finance literature can be viewed as an attempt at formalising this observation. In this paper we have proposed a simple method of combining asymmetric cointegration with a dynamically flexible ARDL model and have derived the associated error correction framework. The desirable features of the NARDL model are threefold. Firstly, the estimation of the ECM in one step is likely to improve the performance of the model in small samples, particularly in terms of the power of the cointegration tests. Secondly, the ability to simultaneously estimate both long- and short-run asymmetries in a computationally simple and tractable manner reflects the flexibility of our modelling approach. Moreover, our technique provides a straightforward means of testing both long- and short-run symmetry restrictions. Finally, the use of asymmetric dynamic multipliers provides an intuitive and computationally straightforward means of assessing the traverse between the short- and long-run, a result with significant theoretical appeal. While the dynamic adjustment in most ECMs is discussed in terms of the percentage of the disequilibrium error that is corrected in each period, our approach sheds light on the nature of this dynamic adjustment, mapping the gradual movement of the process under scrutiny from initial equilibrium through the shock and toward the new equilibrium.

These key strengths of the NARDL framework have been demonstrated in the case of the long- and short-run asymmetry of the unemployment-output relationship and the short-run asymmetry characterising retail gasoline price adjustments. The results suggest that the imposition of long-run symmetry where the underlying relationship is nonlinear will confound efforts to test for the existence of a stable long-run relationship and will result in spurious dynamic responses. Similarly, our results stress the importance of correctly capturing short-run asymmetries in order to illuminate potentially important differences in the response of economic agents to positive and negative shocks.

In summary, NARDL represents the simplest method of modelling combined short- and long-run asymmetries yet developed. At this point, it seems appropriate to mention three obvious extensions which present themselves. Firstly, the model can be related to the threshold literature by generalising to the case of one or more unknown non-zero thresholds for use in the construction of the partial sum processes. This is the subject of ongoing research by Greenwood-Nimmo, Shin and Van Treeck (2011), in which we employ Hansen's (2000) approach to estimation and inference in models with unknown threshold parameters. One could further extend research in this vein by allowing for the state-contingency of the error correction term, ρ (*i.e.* distinguishing between ρ^{+} and ρ^{-}). Secondly, although highly challenging, the development of a system equivalent of our model capable of dealing with multiple long-run relationships would permit the analysis of a more diverse range of macroeconomic phenomena. Finally, the extension of the model to the dynamic heterogeneous panel context may broaden its appeal further still. The obvious starting point for such developments is the pooled mean group framework advanced by Pesaran, Shin and Smith (1999), which is readily estimable by FIML under the assumption of long-run homogeneity.

Acknowledgments

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Notes

[1]The present version of the paper is a substantially revised version of Shin and Yu (2004), which has benefited greatly from a sequence of incremental improvements and additions arising from the constructive comments of conference and seminar participants and from editorial feedback. Earlier versions of the paper circulated under the titles *"An ARDL Approach to an Analysis of Asymmetric Long-run Cointegrating Relationships"* and *"Modelling Asymmetric Cointegration and Dynamic Multipliers in an ARDL Framework"*. By virtue of its wide circulation and prolonged availability as a working paper, our research has informed the development of a subsequent literature that we now discuss. In all cases, however, the development of the NARDL model is properly credited.

[2]The presence of long-run asymmetry will induce a ratchet mechanism if the respective positive and negative regime probabilities are approximately equal and the shocks under each regime are of comparable magnitude. In the more general case in which these conditions are not satisfied, no such simple conclusion may be drawn.

[3]Consider the threshold ECM as an example, in which case the choice of the transition variable is of importance both theoretically and empirically. In general, the asymptotic distribution of the test statistic for the null of linearity or symmetry is not only non-standard but also depends on these transition variables.

[4]The concept of asymmetric cointegration is easily conceptualised by use of a simple example. Consider the output-unemployment relationship. In a standard cointegrating regression, one models y_t and x_t subject to a common stochastic trend. As this relationship

is assumed to hold in the long-run, it represents the equilibrium to which the system returns after a perturbation (*i.e.* it acts as a global attractor). However, in our framework, the long-run relationship between y_t and x_t is modelled as piecewise linear subject to the decomposition of x_t. Suppose that $|\beta^+| < |\beta^-|$ in (2.1). This suggests that the long-run effect of a unit negative change in output will increase unemployment by a greater amount than a unit positive change would reduce it. Thus, our model includes a regime-switching cointegrating relationship in which regime transitions are governed by the sign of Δx_t. The economic implication of this line of reasoning is that equilibrium need not be unique in a globally linear sense. The link to the path dependency literature is apparent.

[5] In the special case where v_t is normally distributed with zero mean and constant variance σ_v^2, it is well-established that the censored normal variates, $v_t^+ = \max[0, v_t]$ and $v_t^- = \min[0, v_t]$, will have $E\left(v_t^+\right) = \frac{\sigma_v}{\sqrt{2\pi}}$, $E\left(v_t^-\right) = -\frac{\sigma_v}{\sqrt{2\pi}}$, and $Var\left(v_t^+\right) = Var\left(v_t^-\right) = \frac{\sigma_v^2}{2}\frac{\pi-1}{\pi}$. We are grateful to Jinseo Cho for pointing this issue out and encouraging us to provide a more general result in Theorem 1.

[6]Notice that the analysis of short-run dynamic asymmetries is not straightforward in the context of the static regression model employing the semiparametric approach.

[7]In some cases, most notably where the growth rates of the series in $\boldsymbol{x}_t$ are predominantly positive (negative), the use of a zero threshold may result in one regime containing an undesirably low number of effective observations. In such situations, an obvious candidate for an alternative threshold is the mean growth rate.

[8]For convenience we employ the same lag order, q. One may also allow for feedback effects from the lagged Δy's on Δx_t in (2.8).

[9]While the associated critical values can be tabulated easily using stochastic simulation, it is impractical to provide a meaningful set of critical values covering all possible combinations. It is generally straightforward, however, to compute the appropriate p-values by means of standard bootstrap techniques.

[10]It is straightforward to extend similar reasoning to the more general case with multiple regressors decomposed into partial sum processes.

[11]The level parameters are obtained as follows:

$$\phi_1 = \rho + 1 + \varphi_1;\ \phi_i = \varphi_i - \varphi_{i-1},\ i = 2, ..., p-1;\ \phi_p = -\varphi_{p-1};$$

$$\boldsymbol{\theta}_0^\ell = \boldsymbol{\pi}_0^\ell;\ \boldsymbol{\theta}_1^\ell = \boldsymbol{\theta}^\ell - \boldsymbol{\pi}_0^\ell + \boldsymbol{\pi}_1^\ell;\ \boldsymbol{\theta}_i^\ell = \boldsymbol{\pi}_i^\ell - \boldsymbol{\pi}_{i-1}^\ell,\ i = 2, ..., q-1;\ \boldsymbol{\theta}_q^\ell = -\boldsymbol{\pi}_{q-1}^\ell,\ \ell = +, -.$$

[12]The dynamic multipliers, $\boldsymbol{\lambda}_j^+$ and $\boldsymbol{\lambda}_j^-$ for $j = 0, 1, ...,$ can be evaluated using the following recursive relationships in which $\boldsymbol{\lambda}_0^\ell = \boldsymbol{\theta}_0^\ell$, $\phi_j = 0$ for $j < 1$ and $\boldsymbol{\lambda}_j^\ell = \mathbf{0}$ for $j < 0$:

$$\boldsymbol{\lambda}_j^\ell = \phi_1 \boldsymbol{\lambda}_{j-1}^\ell + \phi_2 \boldsymbol{\lambda}_{j-2}^\ell + ... + \phi_{j-1} \boldsymbol{\lambda}_1^\ell + \phi_j \boldsymbol{\lambda}_0^\ell + \boldsymbol{\theta}_j^\ell,\ \ell = +, -,\ j = 1, 2, ...,$$

[13]The final specification in Borenstein *et al.* (1997) differs slightly from (2.14) as the lagged Δy_t's on the right hand side are also decomposed into positive and negative changes. However, their derivation is rather *ad hoc.*

[14]Short-run symmetry restrictions (especially the pair-wise restrictions) may be excessively restrictive in many applications although they may be useful in providing more precise estimation results, particularly when estimating a long-run asymmetric relationship in small samples. The additive symmetry restrictions are somewhat weaker and have been discussed in the literature in terms of assessing the validity of the liquidity constraint

where $\sum_{i=0}^{q-1} \boldsymbol{\pi}_i^+ < \sum_{i=0}^{q-1} \boldsymbol{\pi}_i^-$ (*e.g.* Van Treeck, 2008).

[15]Webber (2000) utilises a similar approach in his analysis of the asymmetric pass-through from exchange rates, decomposed as the partial sum processes of appreciations and depreciations, to import prices.

[16]Full results are available on request.

[17]We employ a non-parametric bootstrapping routine and use 50,000 replications after rejecting those for which $\rho > -1 \times 10^{-4}$. Full details are available on request.

[18]Further examples of the use of positive/negative decompositions in the modelling of asymmetry in the unemployment-output relationship include Lee (2000) and Virén (2001).

[19]Seasonally-adjusted monthly data for unemployment and industrial production covering the range 1982m2-2003m11 were collected from the OECD's Main Economic Indicators. Although not presented here, ADF testing lends overwhelming support to the hypothesis that all variates are I(1).

[20]Bachmeier and Griffin (2003) criticise BCG for their use of 'nonstandard estimation methodology' and low-frequency data, arguing that the two-step EG method finds no evidence of asymmetry and, moreover, that the BCG method finds no evidence of asymmetry when applied to their daily dataset. While there is some debate over the optimal data frequency for the study of price shocks, the criticism of the one-step BCG estimation process is unwarranted (*c.f.* Pesaran and Shin, 1998). Indeed, as noted above, estimating the ECM in a single step yields superior performance in small samples, particularly in relation to the power of the cointegration tests.

[21]An early and notable paper combining both short- and long-run asymmetries in the analysis of the nonlinearities characterising the relationship between upstream and downstream prices in the oil industry is Balke, Brown, and Yücel (1998). The authors extend BCG's modelling to incorporate long-run nonlinearities and find evidence of "pervasive and large" asymmetries in all cases apart from their levels specification (p. 10).

[22]As discussed above, in the presence of weakly endogenous regressors and/or serially correlated errors, the OLS estimator in the first step remains consistent but is inefficient. Furthermore, if the AR coefficients are significantly different from zero, the OLS estimator becomes inconsistent and is thus poorly determined in finite samples (see Pesaran and Shin, 1998, and Pesaran *et al.*, 2001).

[23]The Dubai spot price (US$/bbl), p_t^o, was retrieved from the Korean Energy Economics Institute (`http://www.keei.re.kr`) prior to 1998 and from PETRONET (`http://www.petronet.co.kr`) thereafter. The gasoline price index (2000Y=100), p_t, and the KRW/USD exchange, x_t, were retrieved from the Economic Statistics System of the Bank of Korea. All data is in logarithmic form.

[24]The Engle-Granger residual-based test associated with the static linear regression of price on a constant, time trend, oil price and exchange rate (all in logs) returns a maximum value of -3.91 compared to a 5% critical value of -4.32. Similarly, for the static asymmetric regression of price on p^{o+}, p^{o-}, x^+ and x^-, as well as a constant and a trend, $EG_{MAX} = -4.57$ compared to a 5% critical value of -5.01.

A Appendix: Proof of Theorem 1

The OLS estimator, $\hat{\boldsymbol{\beta}} := (\hat{\beta}^{+}, \hat{\beta}^{-})'$, in (2.1) is obtained by

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} \sum_{t=1}^{T} \left(x_t^{+}\right)^2 & \sum_{t=1}^{T} x_t^{+} x_t^{-} \\ \sum_{t=1}^{T} x_t^{+} x_t^{-} & \sum_{t=1}^{T} \left(x_t^{-}\right)^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{t=1}^{T} x_t^{+} y_t \\ \sum_{t=1}^{T} x_t^{-} y_t \end{bmatrix},$$

so that

$$\hat{\boldsymbol{\beta}} - \boldsymbol{\beta} = \frac{1}{D_T} \begin{bmatrix} \sum_{t=1}^{T} \left(x_t^{-}\right)^2 & -\sum_{t=1}^{T} x_t^{+} x_t^{-} \\ -\sum_{t=1}^{T} x_t^{+} x_t^{-} & \sum_{t=1}^{T} \left(x_t^{+}\right)^2 \end{bmatrix} \begin{bmatrix} \sum_{t=1}^{T} x_t^{+} u_t \\ \sum_{t=1}^{T} x_t^{-} u_t \end{bmatrix} = \frac{1}{D_T} \begin{bmatrix} A_T \\ B_T \end{bmatrix},$$

where $D_T := \sum_{t=1}^{T} \left(x_t^{+}\right)^2 \sum_{t=1}^{T} \left(x_t^{-}\right)^2 - \left(\sum_{t=1}^{T} x_t^{+} x_t^{-}\right)^2$, $A_T := \sum_{t=1}^{T} \left(x_t^{-}\right)^2 \sum_{t=1}^{T} x_t^{+} u_t - \sum_{t=1}^{T} x_t^{+} x_t^{-} \sum_{t=1}^{T} x_t^{-} u_t$, and $B_T := -\sum_{t=1}^{T} x_t^{+} x_t^{-} \sum_{t=1}^{T} x_t^{+} u_t + \sum_{t=1}^{T} \left(x_t^{+}\right)^2 \sum_{t=1}^{T} x_t^{-} u_t$. We now let

$$w_t^{+} := \max[0, v_t] - \mu^{+}, \quad w_t^{-} := \min[0, v_t] - \mu^{-},$$

where $\mu^{+} := E\left[\max[0, v_t]\right]$ and $\mu^{-} := E\left[\min[0, v_t]\right]$, so that

$$x_t^{+} \equiv t\mu^{+} + \sum_{j=1}^{t} w_j^{+}, \quad x_t^{-} \equiv t\mu^{-} + \sum_{j=1}^{t} w_j^{-}$$

Hence, we obtain:

$$\begin{aligned} D_T = & \left\{\sum_{t=1}^{T} t^2\right\} \left\{\sum_{t=1}^{T} \left[\mu^{+2} \left(\sum_{j=1}^{t} w_j^{-}\right)^2 + \mu^{-2} \left(\sum_{j=1}^{t} w_j^{+}\right)^2 - 2\mu^{+}\mu^{-} \left(\sum_{j=1}^{t} w_j^{-}\right) \left(\sum_{j=1}^{t} w_j^{+}\right)\right]\right\} \\ & - \left\{\mu^{+2} \left(\sum_{t=1}^{T} t \sum_{j=1}^{t} w_j^{-}\right)^2 + \mu^{-2} \left(\sum_{t=1}^{T} t \sum_{j=1}^{t} w_j^{+}\right)^2 - 2\mu^{+}\mu^{-} \left(\sum_{t=1}^{T} t \sum_{j=1}^{t} w_j^{-}\right) \left(\sum_{t=1}^{T} t \sum_{j=1}^{t} w_j^{+}\right)\right\} \\ & + o_P(T^5). \end{aligned}$$

Here, $o_P(T^6)$ terms are canceled off, and the remaining next-order terms are stated as above. We now note that

$$\frac{1}{T^3}\sum_{t=1}^{T} t^2 = \frac{1}{3} + o(1),$$

$$\mu^{+2}\left(\sum_{j=1}^{t} w_j^-\right)^2 + \mu^{-2}\left(\sum_{j=1}^{t} w_j^+\right)^2 - 2\mu^+\mu^-\left(\sum_{j=1}^{t} w_j^-\right)\left(\sum_{j=1}^{t} w_j^+\right) = \left(\sum_{j=1}^{t} s_j\right)^2$$

where $s_j \equiv \mu^+ w_j^- - \mu^- w_j^-$ by the definitions of w_j^- and w_j^+. Hence, by Donsker's FCLT.

$$T^{-1/2}\sum_{j=1}^{T(\cdot)} s_t/\sigma_s \Rightarrow W_{\tilde{s}}(\cdot),$$

where $\sigma_s^2 := Var\,(s_t)$, $\Rightarrow$ indicates weak convergence, and $W_{\tilde{s}}(r)$ is the standard Brownian motions defined on $r \in [0, 1]$. Therefore,

$$T^{-2}\sum_{t=1}^{T}\left(\sum_{j=1}^{t} s_j\right)^2 \Rightarrow \sigma_s^2 \int_0^1 W_{\tilde{s}}(r)^2 dr$$

by the CMT (*e.g.* Eq. (17.3.22) of Hamilton (1994), p. 486). Also notice that

$$\mu^{+2}\left(\sum_{t=1}^{T} t\sum_{j=1}^{t} w_j^-\right)^2 + \mu^{-2}\left(\sum_{t=1}^{T} t\sum_{j=1}^{t} w_j^+\right)^2 - 2\mu^+\mu^-\left(\sum_{t=1}^{T} t\sum_{j=1}^{t} w_j^-\right)\left(\sum_{t=1}^{T} t\sum_{j=1}^{t} w_j^+\right)$$
$$= \left(\sum_{t=1}^{T} t\sum_{j=1}^{t}\left(\mu^+ w_j^- - \mu^- w_j^+\right)\right)^2 = \left(\sum_{t=1}^{T} t\sum_{j=1}^{t} s_j\right)^2,$$

then it follows that

$$T^{-\frac{5}{2}}\sum_{t=1}^{T} t\sum_{j=1}^{t} s_j \Rightarrow \sigma_s \int_0^1 r W_{\tilde{s}}(r) dr$$

by the CMT. Collecting all these results we obtain:

$$T^{-5}D_T \Rightarrow \sigma_s^2 \left[\frac{1}{3} \int_0^1 W_{\tilde{s}}(r)^2 dr - \left(\int_0^1 r W_{\tilde{s}}(r) dr \right)^2 \right]. \tag{A.1}$$

Next, we consider the asymptotic weak limit of the numerator of $\hat{\beta}^+ - \beta^+$. For this, we note that $O_P(T^{9/2})$ terms cancel off, so that the remaining next-order terms are $O_p(T^4)$, so that

$$\begin{aligned}
A_T :=& \sum_{t=1}^T (x_t^-)^2 \sum_{t=1}^T x_t^+ u_t - \sum_{t=1}^T x_t^+ x_t^- \sum_{t=1}^T x_t^- u_t \\
=& \left\{ \mu^{-2} \sum_{t=1}^T t^2 \sum_{t=1}^T u_t \sum_{j=1}^t w_j^+ + 2\mu^- \mu^+ \sum_{t=1}^T t u_t \sum_{t=1}^T \sum_{j=1}^t w_j^- \right\} \\
&- \left\{ \mu^+ \mu^- \sum_{t=1}^T t^2 \sum_{t=1}^T u_t \sum_{j=1}^t w_j^+ + \sum_{t=1}^T t \sum_{j=1}^t (\mu^+ w_j^- + \mu^- w_j^+) \mu^- \sum_{t=1}^T t u_t \right\} + o_P(T^4) \\
=& \mu^- \left\{ - \left(\sum_{t=1}^T t^2 \right) \left(\sum_{t=1}^T u_t \sum_{j=1}^t s_j \right) + \left(\sum_{t=1}^T t \sum_{j=1}^t s_j \right) \left(\sum_{t=1}^T t u_t \right) \right\} + o_P(T^4) \qquad \text{(A.2)}
\end{aligned}$$

where we also employ the definition of $s_j := \mu^+ w_j^- - \mu^- w_j^+$. Then, by the CMT (*e.g.* Eqs. (f) on p. 548 and (17.3.19) on p. 486 of Hamilton (1994), respectively),.we have:

$$T^{-1} \sum_{t=1}^T u_t \sum_{j=1}^t s_j \Rightarrow \sigma_s \sigma_u \int_0^1 W_{\tilde{s}}(r) dW_{\tilde{u}}(r) \tag{A.3}$$

$$T^{-\frac{3}{2}} \sum_{t=1}^T t u_t \Rightarrow \sigma_u \left(W_{\tilde{u}}(1) - \int_0^1 W_{\tilde{u}}(r) dr \right) \tag{A.4}$$

where $W_{\tilde{u}}(\cdot)$ is the standard Brownian motion independent of $W_{\tilde{s}}(\cdot)$. Collecting all these results and (A.4) and plugging them into A_T, we obtain by the CMT:

$$T^{-4}A_T \Rightarrow \mu^{-}\sigma_s\sigma_u \left\{ -\frac{1}{3}\int_0^1 W_{\tilde{s}}(r)dW_{\tilde{u}}(r) + \int_0^1 rW_{\tilde{s}}(r)dr \left(W_{\tilde{u}}(1) - \int_0^1 W_{\tilde{u}}(r)dr \right) \right\} \tag{A.5}$$

We now examine the numerator of $(\hat{\beta}^{-} - \beta^{-})$ in a similar manner. That is,

$$B_T := \mu^{+}\sigma_s\sigma_u \left\{ \left(\sum_{t=1}^{T} t^2 \right) \left(\sum_{t=1}^{T} u_t \sum_{j=1}^{t} s_j \right) - \left(\sum_{t=1}^{T} t \sum_{j=1}^{t} s_j \right) \left(\sum_{t=1}^{T} tu_t \right) \right\} + o_P(T^4), \tag{A.6}$$

and

$$T^{-4}B_T \Rightarrow \mu^{+}\sigma_s\sigma_u \left\{ \frac{1}{3}\int_0^1 W_{\tilde{s}}(r)dW_{\tilde{u}}(r) - \int_0^1 rW_{\tilde{s}}(r)dr \left(W_{\tilde{u}}(1) - \int_0^1 W_{\tilde{u}}(r)dr \right) \right\} \tag{A.7}$$

Combining (A.5) and (A.7) respectively with (A.1) we obtain the main results.

Next, from (A.2) and (A.6), it is easily seen that

$$\mu^{+}A_T + \mu^{-}B_T = o_P(T^4),$$

which proves the final result in Theorem 1.

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Table 1: Monte Carlo Simulation Results: Bias, Standard Error and RMSE of the OLS Estimator

	$T = 100$				$T = 200$				$T = 400$			
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
	α	0.001	0.308	6.932	α	0.001	0.194	3.082	α	0.001	0.130	1.451
	ρ	-0.063	0.070	2.121	ρ	-0.029	0.043	0.825	ρ	-0.014	0.028	0.354
	θ^+	0.019	0.054	1.283	θ^+	0.011	0.028	0.475	θ^+	0.006	0.016	0.195
$\omega = -0.5$	θ^-	0.051	0.073	2.001	θ^-	0.026	0.044	0.806	θ^-	0.013	0.029	0.351
	φ^+	-0.001	0.179	4.019	φ^+	0.000	0.122	1.930	φ^+	-0.001	0.085	0.954
	φ^-	-0.002	0.178	4.011	φ^-	-0.001	0.122	1.937	φ^-	0.001	0.085	0.954
	β^+	-0.031	0.205	4.664	β^+	-0.010	0.102	1.628	β^+	-0.003	0.051	0.567
	β^-	-0.031	0.205	4.661	β^-	-0.010	0.102	1.631	β^-	-0.003	0.051	0.567
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
	α	0.002	0.366	8.230	α	0.001	0.229	3.633	α	0.000	0.150	1.681
	ρ	-0.075	0.077	2.427	ρ	-0.037	0.049	0.970	ρ	-0.018	0.033	0.417
	θ^+	0.037	0.072	1.811	θ^+	0.018	0.036	0.647	θ^+	0.009	0.020	0.251
$\omega = 0$	θ^-	0.075	0.098	2.773	θ^-	0.037	0.056	1.062	θ^-	0.018	0.035	0.441
	φ^+	0.002	0.206	4.631	φ^+	0.000	0.141	2.228	φ^+	0.001	0.098	1.102
	φ^-	-0.001	0.205	4.613	φ^-	0.000	0.140	2.221	φ^-	0.000	0.099	1.104
	β^+	-0.002	0.227	5.104	β^+	0.000	0.114	1.812	β^+	0.000	0.057	0.637
	β^-	-0.001	0.227	5.097	β^-	0.000	0.114	1.813	β^-	0.000	0.057	0.638
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
	α	0.005	0.311	7.001	α	0.001	0.195	3.096	α	0.001	0.129	1.441
	ρ	-0.063	0.070	2.121	ρ	-0.029	0.043	0.826	ρ	-0.014	0.028	0.354
	θ^+	0.044	0.074	1.929	θ^+	0.018	0.036	0.634	θ^+	0.008	0.019	0.234
$\omega = 0.5$	θ^-	0.075	0.102	2.854	θ^-	0.032	0.054	1.001	θ^-	0.015	0.032	0.396
	φ^+	0.002	0.178	4.001	φ^+	0.001	0.123	1.948	φ^+	0.000	0.085	0.952
	φ^-	0.002	0.178	4.002	φ^-	0.000	0.122	1.933	φ^-	0.000	0.085	0.957
	β^+	0.031	0.207	4.714	β^+	0.010	0.102	1.625	β^+	0.003	0.050	0.565
	β^-	0.032	0.207	4.714	β^-	0.010	0.102	1.621	β^-	0.003	0.050	0.566

Note: Bias = $\hat{\theta}_R - \theta_0$, where θ_0 is the true value of the coefficient θ and $\hat{\theta}_R$ is the mean of the estimates of θ across replications, *i.e.*, $\hat{\theta}_R = \sum_{i=1}^{R} \hat{\theta}_i / R$, where R is the number of replications (we set $R = 3,000$ in all cases). STDE θ denotes the standard error of the estimator, $\hat{\theta}_i$, across replications. RMSE denotes the root mean squared error of $\hat{\theta}_i$, defined as $\sqrt{R^{-1} \sum_{i=1}^{R} (\hat{\theta}_i - \theta_0)^2}$.

Table 2: Monte Carlo Simulation Results: Size and Power of Wald and PSS Tests

	$T = 100$			$T = 200$			$T = 400$		
	Test	Power	Size	Test	Power	Size	Test	Power	Size
	W_{LR}	0.981	0.089	W_{LR}	1.000	0.067	W_{LR}	1.000	0.059
$\omega = -0.5$	W_{SR}	0.425	0.075	W_{SR}	0.675	0.050	W_{SR}	0.935	0.055
	$F_{PSS}^{k=1}$	0.610	0.040	$F_{PSS}^{k=1}$	0.995	0.045	$F_{PSS}^{k=1}$	1.000	0.030
	$F_{PSS}^{k=2}$	0.765	0.090	$F_{PSS}^{k=2}$	1.000	0.100	$F_{PSS}^{k=2}$	1.000	0.070
	$F_{PSS}^{(b)}$	0.720	0.050	$F_{PSS}^{(b)}$	1.000	0.050	$F_{PSS}^{(b)}$	1.000	0.050
	Test	Power	Size	Test	Power	Size	Test	Power	Size
	W_{LR}	0.974	0.100	W_{LR}	1.000	0.075	W_{LR}	1.000	0.062
$\omega = 0$	W_{SR}	0.308	0.051	W_{SR}	0.548	0.053	W_{SR}	0.870	0.035
	$F_{PSS}^{k=1}$	0.329	0.036	$F_{PSS}^{k=1}$	0.947	0.030	$F_{PSS}^{k=1}$	1.000	0.025
	$F_{PSS}^{k=2}$	0.527	0.080	$F_{PSS}^{k=2}$	0.988	0.072	$F_{PSS}^{k=2}$	1.000	0.070
	$F_{PSS}^{(b)}$	0.422	0.050	$F_{PSS}^{(b)}$	0.976	0.050	$F_{PSS}^{(b)}$	1.000	0.050
	Test	Power	Size	Test	Power	Size	Test	Power	Size
	W_{LR}	0.982	0.098	W_{LR}	1.000	0.075	W_{LR}	1.000	0.061
$\omega = 0.5$	W_{SR}	0.385	0.075	W_{SR}	0.675	0.025	W_{SR}	0.925	0.055
	$F_{PSS}^{k=1}$	0.540	0.035	$F_{PSS}^{k=1}$	0.985	0.025	$F_{PSS}^{k=1}$	1.000	0.025
	$F_{PSS}^{k=2}$	0.735	0.080	$F_{PSS}^{k=2}$	1.000	0.080	$F_{PSS}^{k=2}$	1.000	0.075
	$F_{PSS}^{(b)}$	0.655	0.050	$F_{PSS}^{(b)}$	0.995	0.050	$F_{PSS}^{(b)}$	1.000	0.050

Note: W_{LR} denotes the Wald test of the null hypothesis of long-run symmetry defined as $\theta^+ = \theta^-$. W_{SR} is the Wald test of the short-run symmetry restrictions $\varphi^+ = \varphi^-$. $F_{PSS}^{k=n}$ denotes the PSS F-test of the null hypothesis $\rho = \theta^+ = \theta^- = 0$ using the $k = n$ critical values where $n = (1, 2)$ for the case where all regressors follow nonstationary $I(1)$ processes. $F_{PSS}^{(b)}$ refers to the bootstrapped PSS test.

Table 3: Static Estimation of the Unemployment-Output Relationship

(a) Static Linear Regression

	US		Canada		Japan	
Var.	Coeff.	S.E.	Coeff.	S.E.	Coeff.	S.E.
Constant	73.16	3.92	74.96	2.94	29.94	1.25
Trend	0.03	0.00	0.03	0.00	0.02	0.00
y_t	-15.66	0.94	-15.19	0.70	-6.38	0.28
R^2	0.77		0.78		0.89	
Adj. R^2	0.77		0.78		0.89	
χ^2_{SC}	250.84[.000]		233.28[.000]		235.08[.000]	
χ^2_{H}	69.29[.000]		1.95[.163]		0.29[.593]	
χ^2_{FF}	109.11[.000]		0.21[.901]		60.72[.000]	
χ^2_{N}	3.40[.183]		6.52[.011]		21.62[.000]	
EG_{MAX}	-2.90		-2.42		-2.86	

(b) Static Asymmetric Regression

	US		Canada		Japan	
Var.	Coeff.	S.E.	Coeff.	S.E.	Coeff.	S.E.
Const.	7.82	0.10	10.56	0.10	2.55	0.62
y_t^+	-10.73	0.51	-13.05	0.48	-4.61	0.28
y_t^-	-25.83	1.81	-20.38	0.92	-7.70	0.33
R^2	0.78		0.81		0.87	
Adj. R^2	0.77		0.81		0.87	
χ^2_{SC}	248.82[.000]		231.04[.000]		240.02[.000]	
χ^2_{H}	66.99[.000]		0.31[.580]		0.16[.690]	
χ^2_{FF}	110.39[.000]		0.23[.892]		57.18[.000]	
χ^2_{N}	11.23[.004]		7.97[.005]		22.69[.000]	
$W_{y^+=y^-}$	129.20[.000]		258.10[.000]		1607.50[.000]	
EG_{MAX}	-2.79		-2.60		-2.55	

Note: y_t denotes the natural logarithm of industrial production and y_t^+ and y_t^- the associated positive and negative partial sum processes. Note also that in order to accommodate the strong trending behavior of y_t, we include a deterministic time trend in the symmetric case. χ^2_{SC}, χ^2_H, χ^2_{FF} and χ^2_N denote LM tests for serial correlation, heteroscedasticity, functional form (Ramsey's RESET test) and normality, respectively. Figures in square parentheses are the associated p-values. $W_{y^+=y^-}$ denotes the Wald test of the equality of the coefficients associated with y_t^+ and y_t^-. EG_{MAX} denotes the largest value of the Engle-Granger residual-based ADF test. The 95% critical values of the EG test are -3.42 (panel (a)) and -3.77 (panel (b)).

Table 4: Dynamic Linear Estimation of the Unemployment-Output Relationship

	US			Canada			Japan	
Var.	Coeff.	S.E.	Var.	Coeff.	S.E.	Var.	Coeff.	S.E.
u_{t-1}	-0.03	0.01	u_{t-1}	-0.02	0.01	u_{t-1}	0.00	0.01
y_{t-1}	-0.04	0.07	y_{t-1}	-0.09	0.10	y_{t-1}	-0.02	0.06
Δu_{t-1}	-0.17	0.06	Δu_{t-2}	-0.12	0.06	Δu_{t-1}	-0.26	0.06
Δu_{t-11}	0.13	0.05	Δy_t	-4.40	1.19	Δu_{t-2}	-0.22	0.06
Δy_t	-8.17	1.61	Δy_{t-2}	-2.83	1.21	Δu_{t-10}	0.16	0.06
Δy_{t-2}	-4.73	1.58	Δy_{t-6}	-3.01	1.16	Δu_{t-12}	-0.18	0.06
Δy_{t-4}	-4.04	1.50	Const.	0.57	0.55	Δy_{t-1}	-1.37	0.42
Const.	0.38	0.35				Δy_{t-2}	-1.27	0.45
						Δy_{t-3}	-1.30	0.43
						Δy_{t-9}	-1.16	0.39
						Const.	0.09	0.27
L_y	-1.66	2.03	L_y	-5.68	3.89	L_y	5.57	20.88
R^2	0.29		R^2	0.13		R^2	0.23	
$\bar{R}^2$	0.27		$\bar{R}^2$	0.11		$\bar{R}^2$	0.20	
χ^2_{SC}	10.75[.550]		χ^2_{SC}	9.35[.673]		χ^2_{SC}	11.95[.450]	
χ^2_{FF}	1.94[.163]		χ^2_{FF}	0.26[.609]		χ^2_{FF}	0.03[.867]	
χ^2_{NOR}	3.72[.156]		χ^2_{NOR}	12.35[.002]		χ^2_{NOR}	0.92[.632]	
χ^2_{HET}	15.19[.000]		χ^2_{HET}	0.09[.770]		χ^2_{HET}	0.41[.521]	
t_{BDM}	-2.34[.136]		t_{BDM}	-1.27[.820]		t_{BDM}	0.57[1.000]	
F_{PSS}	4.69[.081]		F_{PSS}	0.81[.927]		F_{PSS}	0.18[.890]	

Note: u_t denotes the rate of unemployment, measured in percentage points. Here we follow the general-to-specific approach to select the final ARDL specification. The preferred specification is chosen by starting with $\max p = \max q = 12$ and dropping all insignificant stationary regressors. t_{BDM} is the BDM t-statistic while F_{PSS} denotes the PSS F-statistic testing the null hypothesis $\rho = \theta = 0$. The long-run coefficient L_y is defined by $\hat{\beta} = -\hat{\theta}/\hat{\rho}$. Pesaran, Shin and Smith (2001) tabulate the 5% critical values for $k = 1$ as follows: $t_{crit} = -3.22$; $F_{crit} = 5.73$. Empirical p-values are quoted for the BDM t-statistic and the PSS F-statistic.

Table 5: Dynamic Asymmetric Estimation of the Unemployment-Output Relationship

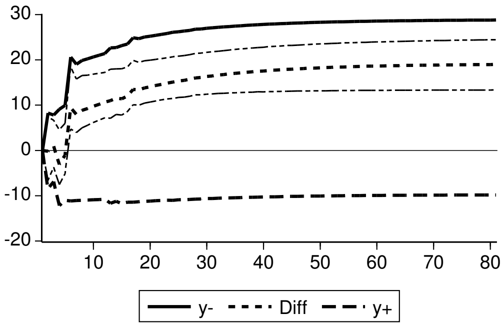
	US			Canada			Japan	
Var.	Coeff.	S.E.	Var.	Coeff.	S.E.	Var.	Coeff.	S.E.
u_{t-1}	-0.06	0.01	u_{t-1}	-0.07	0.02	u_{t-1}	-0.05	0.01
y^+_{t-1}	-0.55	0.17	y^+_{t-1}	-1.27	0.28	y^+_{t-1}	-0.34	0.10
y^-_{t-1}	-1.62	0.50	y^-_{t-1}	-2.09	0.46	y^-_{t-1}	-0.53	0.14
Δu_{t-1}	-0.19	0.06	Δu_{t-2}	-0.13	0.06	Δu_{t-1}	-0.23	0.06
Δu_{t-11}	0.11	0.05	Δu_{t-12}	-0.12	0.06	Δu_{t-2}	-0.19	0.06
Δy^+_t	-8.42	2.23	Δy^+_t	-5.24	1.86	Δu_{t-10}	0.13	0.06
Δy^+_{t-2}	-4.82	1.99	Δy^+_{t-3}	3.69	1.86	Δu_{t-12}	-0.22	0.06
Δy^-_t	-8.24	4.28	Δy^-_t	-5.15	2.60	Δy^+_{t-1}	-1.61	0.65
Δy^-_{t-4}	-9.74	3.77	Δy^-_{t-3}	-5.89	2.64	Δy^+_{t-9}	-1.71	0.66
Const.	0.38	0.11	Const.	0.72	0.19	Δy^-_t	-1.80	0.71
						Const.	0.16	0.04
L_{y^+}	-9.76	1.74	L_{y^+}	-17.26	2.15	L_{y^+}	-7.28	1.64
L_{y^-}	-28.88	6.33	L_{y^-}	-28.48	4.04	L_{y^-}	-11.26	1.97
R^2	0.32		R^2	0.20		R^2	0.24	
$\bar{R}^2$	0.30		$\bar{R}^2$	0.17		$\bar{R}^2$	0.21	
χ^2_{SC}	9.23[.683]		χ^2_{SC}	8.11[.777]		χ^2_{SC}	11.85[.458]	
χ^2_{FF}	0.53[.466]		χ^2_{FF}	9.74[.002]		χ^2_{FF}	0.11[.744]	
χ^2_{NOR}	1.79[.409]		χ^2_{NOR}	12.62[.002]		χ^2_{NOR}	0.30[.861]	
χ^2_{HET}	12.81[.000]		χ^2_{HET}	0.38[.537]		χ^2_{HET}	2.77[.096]	
t_{BDM}	-3.97[.007]		t_{BDM}	-4.12[.006]		t_{BDM}	-3.34[.033]	
F_{PSS}	6.98[.010]		F_{PSS}	7.13[.005]		F_{PSS}	5.38[.038]	
W_{LR}	16.33[.000]		W_{LR}	32.49[.000]		W_{LR}	76.69[.000]	
W_{SR}	0.46[.498]		W_{SR}	3.65[.056]		W_{SR}	2.35[.125]	

Note: L_{y^+} and L_{y^-} denote the long-run coefficients associated with positive and negative changes of output, respectively. W_{LR} refers to the Wald test of long-run symmetry (*i.e.* $L_{y^+} = L_{y^-}$) while W_{SR} denotes the Wald test of the additive short-run symmetry condition. Pesaran, Shin and Smith (2001) tabulate the 5% critical values of t_{BDM} as -3.53 and -3.22 for $k = 2$ and $k = 1$, respectively, while the equivalent values for F_{PSS} are 4.85 and 5.73. Empirical p-values are reported for both tests.

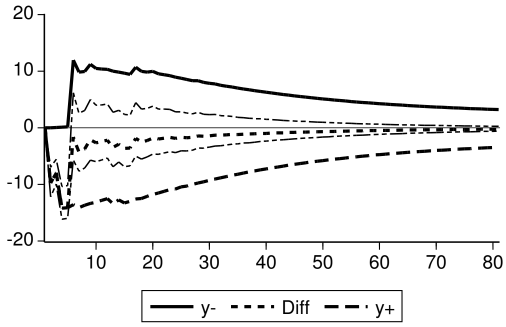
Table 6: Dynamic Asymmetric Estimation of Gasoline Price Adjustments

LR & SR Symmetry			LR Sym & SR Asym			LR & SR Asymmetry		
Var.	Coeff.	S.E.	Var.	Coeff.	S.E.	Var.	Coeff.	S.E.
p_{t-1}	-0.20	0.06	p_{t-1}	-0.18	0.05	p_{t-1}	-0.18	0.07
p^o_{t-1}	0.10	0.03	p^o_{t-1}	0.10	0.02	p^{o+}_{t-1}	0.07	0.04
x_{t-1}	0.26	0.08	x_{t-1}	0.18	0.07	p^{o-}_{t-1}	0.07	0.04
Δp^o_t	0.11	0.05	Δp^{o+}_t	0.30	0.08	x^+_{t-1}	0.13	0.07
Δp^o_{t-1}	0.10	0.04	Δp^{o-}_{t-1}	0.11	0.05	x^-_{t-1}	0.09	0.14
Δx_t	0.56	0.09	Δx^+_t	0.61	0.09	Δp^{o+}_t	0.26	0.09
Δx_{t-3}	0.22	0.09	Δx^+_{t-3}	0.33	0.10	Δp^{o+}_{t-1}	0.16	0.08
Const.	-1.22	0.44	Const.	-0.79	0.37	Δx^+_t	0.68	0.11
						Δx^+_{t-3}	0.39	0.10
						Const.	0.70	0.21
L_{p^o}	0.49	0.06	L_{p^o}	0.54	0.06	L_{p^o+}	0.40	0.25
						L_{p^o-}	0.37	0.30
L_x	1.31	0.13	L_x	1.00	0.18	L_{x^+}	0.73	0.29
						L_{x^-}	0.48	0.73
R^2	0.56		R^2	0.60		R^2	0.60	
Adj. R^2	0.50		Adj. R^2	0.56		Adj. R^2	0.54	
χ^2_{SC}	3.08[.544]		χ^2_{SC}	2.56[.638]		χ^2_{SC}	2.00[.736]	
χ^2_{FF}	8.45[.004]		χ^2_{FF}	1.38[.240]		χ^2_{FF}	1.50[.221]	
χ^2_N	4.47[.107]		χ^2_N	15.43[.000]		χ^2_N	11.25[.004]	
χ^2_H	1.70[.193]		χ^2_H	0.00[.995]		χ^2_H	0.00[.979]	
t_{BDM}	-3.57[.076]		t_{BDM}	-4.02[.107]		t_{BDM}	-2.72[.210]	
F_{PSS}	4.86[.100]		F_{PSS}	9.69[.036]		F_{PSS}	5.51[.239]	
						$W_{LR,\ p^o}$	0.03[.866]	
						$W_{LR,\ x}$	0.17[.680]	
			$W_{SR,\ p^o}$	3.49[.062]		$W_{SR,\ p^o}$	17.17[.000]	
			$W_{SR,\ x}$	44.14[.000]		$W_{SR,\ x}$	56.30[.000]	

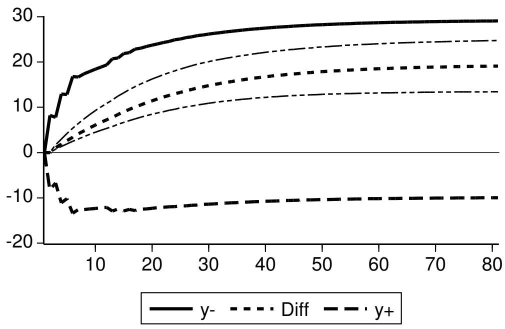
Note: p_t denotes the natural logarithm of the gasoline price index (2000Y=ln(100)), p^o_t denotes the natural logarithm of the price of crude oil (US$/bbl) while x_t denotes the natural logarithm of the KRW/USD exchange rate. The superscripts '+' and '-' denote positive and negative partial sums, respectively. $L_{p^{o+}}$, $L_{p^{o-}}$, L_{x^+} and L_{x^-} denote the long-run coefficients associated with positive and negative changes in the price of crude oil and positive and negative changes in the KRW/USD exchange rate, respectively. $W_{LR,\ p^o}$ refers to the Wald test of the restriction $L_{p^{o+}} = L_{p^{o-}}$ while $W_{LR,\ x}$ refers to the Wald test of $L_{x^+} = L_{x^-}$. $W_{SR,\ p^o}$ and $W_{SR,\ x}$ refer to the Wald tests of the short-run additive symmetry restrictions. The relevant 5% critical values of the t_{BDM} test are -3.99 for $k = 4$ and -3.53 for $k = 2$. Similarly, the critical values of the F_{PSS} test are 4.01 with $k = 4$ and 4.85 with $k = 2$. Empirical p-values are reported for both tests.



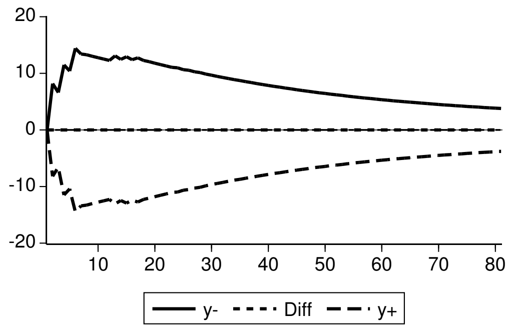
(a) LR & SR asymmetry



(b) LR symmetry & SR asymmetry

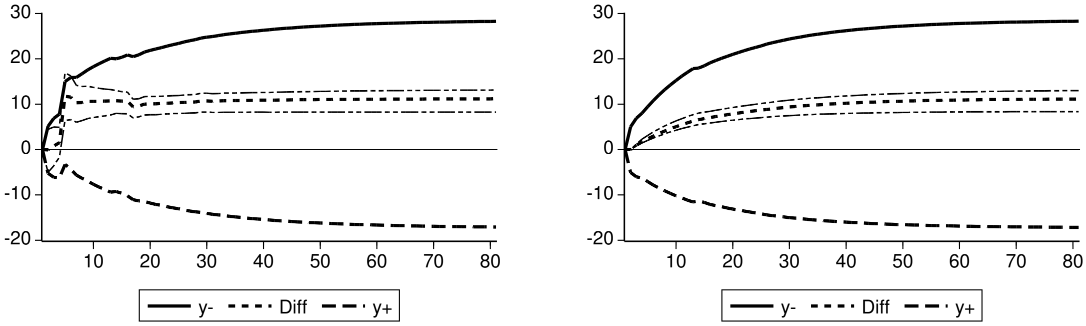


(c) LR asymmetry & SR symmetry



(d) LR & SR symmetry

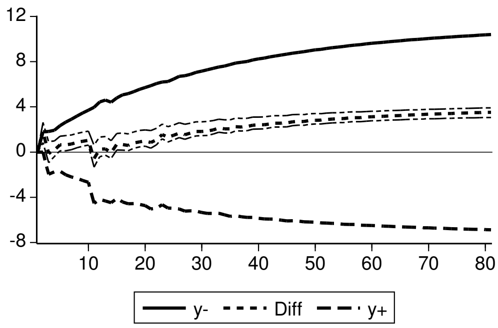
Figure 1: US Unemployment-Output Dynamic Multipliers



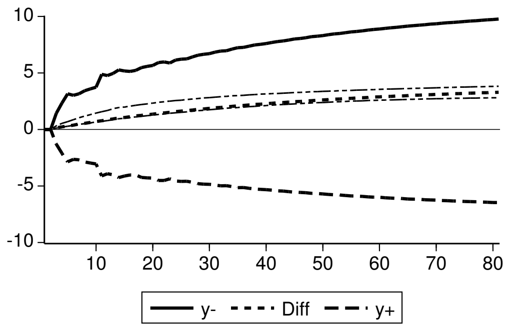
(a) LR & SR asymmetry

(b) LR asymmetry & SR symmetry

Figure 2: Canadian Unemployment-Output Dynamic Multipliers

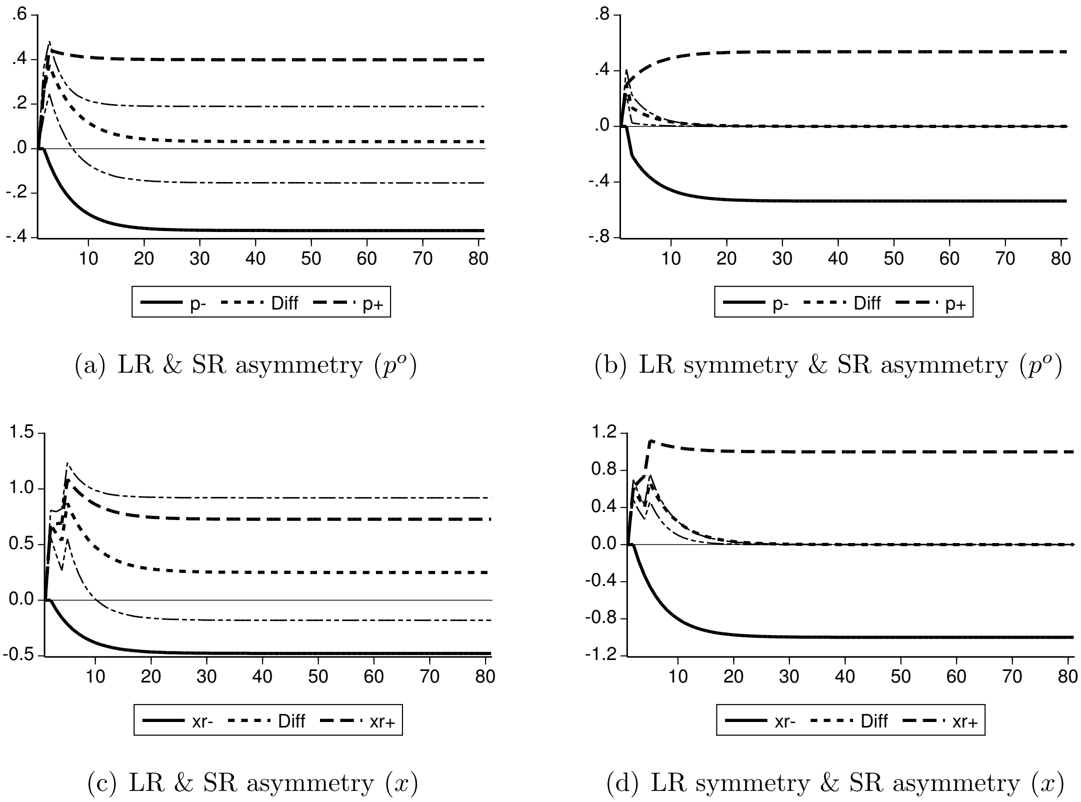


(a) LR & SR asymmetry



(b) LR asymmetry & SR symmetry

Figure 3: Japanese Unemployment-Output Dynamic Multipliers



(a) LR & SR asymmetry (p^o)

(b) LR symmetry & SR asymmetry (p^o)

(c) LR & SR asymmetry (x)

(d) LR symmetry & SR asymmetry (x)

Figure 4: Dynamic Multipliers w.r.t. Oil Price and Exchange Rate Shocks