Advanced Econometrics, Autumn 2011

Assignment 4

To be handed in Friday, October 28.

Carefully describe the steps in any derivation. Refer to theorems and definitions in the curriculum when applying them.

Problem 1

Let $Q_n^*(\theta) = 2(1 + e^{\theta n X_n})^{-1}$ with $\theta \in \Theta = [-5, 5]$ and

$$X_n = \begin{cases} 2 & \text{with probability} \quad \frac{1}{n+1} \\ -2 & \text{with probability} \quad 1 - \frac{1}{n+1}. \end{cases}$$

Find the pointwise (in θ) probability limit of $Q_n^*(\theta)$ for $n \to \infty$. Is the convergence uniform in Θ?

Problem 2

Consider the sequence of functions

$$Q_n(\theta) = \begin{cases} 1 + \theta & \text{for} \quad -1 \leq \theta \leq -1/2 \\ -\theta & \text{for} \quad -1/2 < \theta \leq 0 \\ \frac{\theta^n}{1-\theta^n} & \text{for} \quad 0 < \theta < 1 \\ 0 & \text{for} \quad \theta = 1. \end{cases}$$

1. Check if $Q_n(\theta)$ is uniformly convergent in $\theta \in [-1, 1]$.
2. Check if $\{Q_n(\theta)\}$ is equicontinuous (uniformly continuous in n) in $\theta \in [0, 1/2]$.

Problem 3

Let $\{X_i\}$ be an iid sequence of n Bernoulli distributed random variables with $\Pr(X_i = 1) = \theta$ and $\Pr(X_i = 0) = 1 - \theta$. Assume $\theta \in [\delta, 1 - \delta]$, $0 < \delta < 0.5$.

Derive the maximum likelihood estimator of θ and show, using Theorem 2.1, that it is consistent.

Problem 4

Assume $Q_n(\theta) \to Q_0(\theta)$ uniformly in Θ, where $Q_n(\theta)$ is a continuous and deterministic function of θ. Show that $Q_0(\theta)$ is continuous.

Problem 5

This is a simulation exercise, no proofs needed.

Consider the model

$$Y_i = \beta_1 + \beta_2 X_i + U_i,\ i = 1, .., n \tag{1}$$

where X_i and U_i are independent.

Investigate by simulation whether the ordinary least squares estimator of β_2 is consistent (converges in probability) and whether $\sqrt{n}(\widehat{\beta}_2 - \beta_2)$ converges in distribution. Do this in the following three cases:

1. $U_i \sim \text{iid}\mathcal{N}(0, 1)$ and $X_i \sim \text{iid}\mathcal{N}(0, 2)$,
2. $U_i \sim$iid t-distributed with one degree of freedom and $X_i \sim \text{iid}\mathcal{N}(0, 2)$,
3. $U_i \sim \text{iid}\mathcal{N}(0, 1)$ and $X_i = X_{i-1} + \varepsilon_i$, where $\varepsilon_i \sim \text{iid}\mathcal{N}(0, 1)$.

In all simulations, set $\beta_1 = 0$ and $\beta_2 = 1$ in (1).

Discuss the measures you choose for determining convergence in probability and in distribution.