

Overview

This procedure generates simulated out-of-sample forecasts, forecast errors, and tests of equal mean square error (MSE) and encompassing for a pair of nested models (the first model is a restricted version of the second) of a scalar predictand.

Details on notation, forecast tests, critical values, etc., are available from:

McCracken, Michael W., 2004, "Asymptotics for Out of Sample Tests of Granger Causality," manuscript, University of Missouri. Available at:
<http://www.missouri.edu/~mwmd4f/>

Clark, Todd E., and Michael W. McCracken, 2001, "Tests of Equal Forecast Accuracy and Encompassing for Nested Models," *Journal of Econometrics* 105 (Nov.), pp. 85-110.

Users of the tests of equal MSE should cite the McCracken paper. Users of the critical values for tests of forecast encompassing should cite the Clark and McCracken paper.

An Excel file with complete tables of critical values is available from either
<http://www.missouri.edu/~mwmd4f/>
or
<http://www.kansascityfed.org/econres/staff/tec.htm>

Notation/setup

y_t : scalar random variable to be predicted

$x_{1,t}$ = set of regressors in the restricted model (model 1)

$x_{2,t} = (x'_{1,t}, x'_{22,t})'$: set of regressors in the unrestricted model (model 2)

k_2 : number of variables in model 2 not in model 1

R = number of observations used for estimation of the model from which the first forecast is formed

P = number of forecasts

$\pi = P/R$ (technically, the asymptotics are based on the limit of P/R as both P and R go to infinity). In any given application with given values of P and R , P/R can't equal 0. However, for P small relative to R , some researchers may wish to consider critical values based on $\lim_{P,R \rightarrow \infty} (P/R) = 0$ – that is, use critical values based on an asymptotic approach which differs from that associated with the critical values for $\pi > 0$. As noted below, the asymptotic distributions associated with $\pi = 0$ differ from those associated with $\pi > 0$.

recursive scheme: forecasting models estimated with more data as forecasting moves forward in time

rolling scheme: forecasting models estimated with a moving window of most recent R observations as forecasting moves forward in time

fixed scheme: forecasting models estimated just once, with first R observations; same coefficient estimates used to generate all forecasts

1-step ahead forecast errors from models 1 and 2: $\hat{u}_{1,t+1} = y_{t+1} - x'_{1,t+1}\hat{\beta}_{1,t}$ and $\hat{u}_{2,t+1} = y_{t+1} - x'_{2,t+1}\hat{\beta}_{2,t}$

Note: To simplify notation going forward, for any variable z_{t+1} , let $\sum_t z_{t+1}$ denote the summation $\sum_{t=R}^T z_{t+1}$.

Note: As discussed in Clark and McCracken (2001), among other sources, the tests are one-sided: the only rejections of interest under the alternative are those in the right tail.

Calculation/definition of test statistics for equal MSE

Let $d_{t+1} = \hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2$ and $\bar{d} = P^{-1} \sum_t d_t = MSE_1 - MSE_2$

$$MSE-t = P^{1/2} \frac{\bar{d}}{\sqrt{P^{-1} \sum_t (d_{t+1} - \bar{d})^2}} = P^{1/2} \frac{P^{-1} \sum_t (\hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2)}{\sqrt{P^{-1} \sum_t (\hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2)^2 - \bar{d}^2}}$$

$$MSE-F = P \times \frac{P^{-1} \sum_{t=R}^T (\hat{u}_{1,t+1}^2 - \hat{u}_{2,t+1}^2)}{P^{-1} \sum_{t=R}^T \hat{u}_{2,t+1}^2} = P \times \frac{MSE_1 - MSE_2}{MSE_2}$$

Note: An alternative t-statistic for equal MSE, referred to as MSE-REG by Clark and McCracken (2001), is calculated as the t-statistic associated with the coefficient α from the OLS regression $\hat{u}_{1,t+1} - \hat{u}_{2,t+1} = \alpha(\hat{u}_{1,t+1} + \hat{u}_{2,t+1}) + \text{error term}$. As described in McCracken (2004), the MSE-REG test was proposed by Granger and Newbold (1977). The asymptotic distribution of the MSE-REG test is the same as that of the MSE-t test (i.e., the appropriate critical values are those reported for the MSE-t test).

Calculation/definition of test statistics for forecast encompassing

Let $c_{t+1} = \hat{u}_{1,t+1}(\hat{u}_{1,t+1} - \hat{u}_{2,t+1}) = \hat{u}_{1,t+1}^2 - \hat{u}_{1,t+1}\hat{u}_{2,t+1}$ and $\bar{c} = P^{-1}\sum_t c_t$.

$$\text{ENC-t} = P^{1/2} \frac{\bar{c}}{\sqrt{P^{-1}\sum_t (c_{t+1} - \bar{c})^2}} = P^{1/2} \frac{P^{-1}\sum_t (\hat{u}_{1,t+1}^2 - \hat{u}_{1,t+1}\hat{u}_{2,t+1})}{\sqrt{P^{-1}\sum_t (\hat{u}_{1,t+1}^2 - \hat{u}_{1,t+1}\hat{u}_{2,t+1})^2 - \bar{c}^2}}.$$

$$\text{ENC-NEW} = P \cdot \frac{\bar{c}}{MSE_2} = P \cdot \frac{P^{-1}\sum_t (\hat{u}_{1,t+1}^2 - \hat{u}_{1,t+1}\hat{u}_{2,t+1})}{P^{-1}\sum_t \hat{u}_{2,t+1}^2}.$$

Note: An alternative t-statistic for forecast encompassing, referred to as ENC-REG by Clark and McCracken (2001), is calculated as the t-statistic associated with the coefficient α from the OLS regression $\hat{u}_{1,t+1} = \alpha(\hat{u}_{1,t+1} - \hat{u}_{2,t+1}) + \text{error term}$. The asymptotic distribution of the ENC-REG test is the same as that of the ENC-t test (i.e., the appropriate critical values are those reported for the ENC-t test).